Table 1. Numerical Simulation: Gains from Trade

This table reports numerical and estimation results for the impact of reducing bilateral trade costs, unilateral export costs or unilateral import costs by 20%. Panels A-C show the change in welfare, aggregate productivity, average firm productivity and the covariance of firms' productivity and employment share in different economic environments. In Panels A and B, there is no resource misallocation, and productivity is Pareto or Log-Normal distributed. In Panel C, there is misallocation, and productivity and distortions are joint Log-Normal with σ_η=0.15 and $\rho(\varphi,\eta)$={-0.4,0,0.4}. All other parameter values are as discussed in the text. Panel D reports the estimated effect of increasing export demand or import competition by 20% based on the baseline IV results in Table 5.

	Bilateral Liberalization				Export Liberalization				Import Liberalization			
	Welfare	Agg Prod	Avg Prod	Cov Term	Welfare	Agg Prod	Avg Prod	Cov Term	Welfare	Agg Prod	Avg Prod	Cov Term
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Panel A. No Misallocation (Pareto)												
Flexible *w*	4.76%	4.76%	3.52%	1.23%	1.67%	1.67%	1.23%	0.43%	2.52%	2.52%	1.87%	0.65%
Fixed *w*	3.31%	4.76%	3.52%	1.23%	4.96%	7.16%	5.32%	1.83%	-0.85%	-1.21%	-0.91%	-0.31%
Panel B. No Misallocation (Log-Normal)												
Flexible *w*	3.92%	3.50%	2.75%	0.75%	1.39%	1.22%	0.96%	0.26%	1.95%	1.72%	1.35%	0.37%
Fixed *w*	2.73%	3.50%	2.75%	0.75%	3.77%	4.88%	3.83%	1.05%	-0.49%	-0.60%	-0.48%	-0.12%
Panel C. Misallocation (Joint Log-Normal)												
Flexible *w*												
$\rho=-0.4$	3.92%	3.49%	2.65%	0.84%	1.40%	1.22%	0.92%	0.30%	1.96%	1.72%	1.30%	0.42%
$\rho=0$	3.87%	3.47%	2.80%	0.67%	1.37%	1.21%	0.98%	0.22%	1.93%	1.70%	1.38%	0.32%
$\rho=0.4$	3.85%	3.47%	2.94%	0.53%	1.35%	1.20%	1.04%	0.16%	1.91%	1.70%	1.46%	0.24%
Fixed *w*												
$\rho=-0.4$	-1.68%	-0.05%	-0.16%	0.11%	2.32%	2.26%	1.77%	0.49%	-3.27%	-1.55%	-1.37%	-0.18%
$\rho=0$	2.70%	3.48%	2.81%	0.67%	2.62%	4.46%	3.54%	0.91%	0.58%	-0.21%	-0.13%	-0.08%
$\rho=0.4$	0.92%	7.71%	6.42%	1.29%	0.15%	8.47%	7.11%	1.36%	1.38%	0.03%	0.11%	-0.09%
Panel D. Data												
Estimated Effects (ctry-year FE)						7.96%	5.90%	2.06%		1.36%	1.80%	-0.42%
Estimated Effects (ctry-year & sector-year FE)						7.34%	4.52%	2.82%		10.04%	11.70%	-1.66%

Table 2: Summary Statistics

This table summarizes the variation in aggregate economic activity, aggregate productivity, international trade activity, and institutional and market frictions across countries, sectors and years in the 1998-2011 panel. All variables are defined in the text. The unit of observation is indicated in the panel heading.

	N	Mean	St Dev
Panel A. Country-Sector-Year Level			
ln Output	2,811	8.09	1.77
ln Value Added	2,811	13.51	2.03
ln Employment	2,811	10.21	1.35
ln Exports	2,811	7.65	1.74
ln (Imports - Own-Sector Imp Inputs)	2,811	6.41	1.97
ln Aggregate Productivity	2,811	3.21	1.13
ln Average Productivity	2,811	2.98	1.19
Covariance Term	2,811	0.23	0.22
Δ ln Aggregate Productivity, Δ = 1 year	2,548	0.04	0.10
Δ ln Average Productivity, Δ = 1 year	2,548	0.03	0.09
Δ Covariance Term, Δ = 1 year	2,548	0.01	0.08
Δ ln Aggregate Productivity, Δ = 3 years	2,073	0.11	0.19
Δ ln Average Productivity, Δ = 3 years	2,073	0.09	0.17
Δ Covariance Term, Δ = 3 years	2,073	0.02	0.12
Δ ln Aggregate Productivity, Δ = 5 years	1,587	0.18	0.25
Δ ln Average Productivity, Δ = 5 years	1,587	0.16	0.22
Δ Covariance Term, Δ = 5 years	1,587	0.02	0.14
Panel B. Country(-Year) Level			
Rule of Law	144	1.11	0.49
(Inverse) Corruption	144	1.07	0.69
Labor Market Flexibility	130	3.28	0.37
Creditor Rights Protection	14	5.86	1.79
(Inverse) Product Market Regulation	13	1.17	0.25

Table 3. Trade and Aggregate Performance: OLS Correlation

This table examines the relationship between aggregate economic activity, aggregate productivity and trade exposure at the country-sector-year level. The outcome variable is indicated in the column heading and described in the text. All columns include country-year pair fixed effects, and control for the log number of firms by country-sector-year, the average log number of firms across countries by sector-year, and the average log employment across countries by sector-year. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%, 10%.

	Economic Activity			Aggregate Productivity		
Dep Variable:	ln Output (ikt)	ln Value Added (ikt)	ln Employ-ment (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
	(1)	(2)	(3)	(4)	(5)	(6)
Exp Dem (ikt)	**0.403*** **	**0.380*** **	**0.243*** **	**0.125*** **	**0.080*** **	**0.045*** **
	(0.029)	**(0.022)**	**(0.014)**	**(0.016)**	**(0.016)**	**(0.007)**
Imp Comp (ikt)	**-0.139*** **	**0.041*** **	**-0.066*** **	**0.106*** **	**0.124*** **	**-0.019*** **
	(0.015)	**(0.015)**	**(0.006)**	**(0.013)**	**(0.013)**	**(0.005)**
ln N Firms (ikt)	0.552***	0.573***	0.736***	-0.161***	-0.122***	-0.039***
	(0.023)	(0.023)	(0.019)	(0.020)	(0.018)	(0.007)
Avg ln N Firms (kt)	-0.969***	-0.710***	-0.727***	0.023	0.100***	-0.077***
	(0.032)	(0.033)	(0.023)	(0.033)	(0.033)	(0.010)
Avg ln Employment (kt)	1.285***	0.653***	0.858***	-0.182***	-0.245***	0.063***
	(0.065)	(0.045)	(0.028)	(0.040)	(0.041)	(0.020)
N	2,811	2,811	2,811	2,811	2,811	2,811
R2	0.927	0.928	0.949	0.849	0.868	0.519
Country*Year FE	Y	Y	Y	Y	Y	Y

Table 4. Instrumenting Export Demand and Import Competition: IV First Stage

This table presents the baseline IV first stage. It examines the impact of foreign supply, foreign demand and import tariffs on export and import activity at the country-sector-year level. The outcome variable is indicated in the column heading and described in the text. All columns include country-year pair fixed effects and the full set of controls in Table 3. Columns 2 and 5 (3 and 6) also include sector (sector-year pair) fixed effects. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%, 10%.

Dep Variable:	Exp Dem (ikt)			Imp Comp (ikt)		
	(1)	(2)	(3)	(4)	(5)	(6)
Foreign Demand (ikt)	**0.638***	**0.458***	**0.443***	**-0.002**	**-0.007**	**-0.036**
	(0.034)	**(0.056)**	**(0.062)**	**(0.022)**	**(0.027)**	**(0.030)**
Foreign Supply (ikt)	**0.087***	**0.139**	**0.140***	**0.868***	**0.422***	**0.345***
	(0.015)	**(0.066)**	**(0.081)**	**(0.007)**	**(0.027)**	**(0.031)**
Import Tariff (ikt)	**-4.693***	**0.307**	**0.662**	**-2.802***	**-0.986**	**-1.332***
	(0.847)	**(0.669)**	**(0.816)**	**(0.507)**	**(0.407)**	**(0.437)**
ln N Firms (ikt)	0.555***	0.564***	0.569***	0.036**	0.008	0.007
	(0.034)	(0.032)	(0.032)	(0.018)	(0.016)	(0.016)
Avg ln N Firms (kt)	-0.741***	-0.539***		-0.112***	0.110*	
	(0.033)	(0.134)		(0.025)	(0.062)	
Avg ln Employment (kt)	0.344***	0.490***		0.113***	-0.042	
	(0.065)	(0.089)		(0.042)	(0.055)	
N	2,777	2,777	2,777	2,777	2,777	2,777
R2	0.889	0.921	0.924	0.974	0.985	0.986
Country*Year FE	Y	Y	Y	Y	Y	Y
Sector FE	N	Y	N	N	Y	N
Sector*Year FE	N	N	Y	N	N	Y

Table 5. Impact of Trade on Aggregate Productivity: IV Second Stage

This table presents the baseline IV second stage. It examines the impact of instrumented export demand and import competition on aggregate productivity at the country-sector-year level. The outcome variable is indicated in the column heading and described in the text. All columns include country-year pair fixed effects and the full set of controls in Table 3. Columns 4-6 (7-9) also include sector (sector-year pair) fixed effects. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%, 10%.

Dep Variable:	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
^Exp Dem (ikt)	**0.398*** **	**0.295*** **	**0.103*** **	**0.300*** **	**0.197** **	**0.103** **	**0.367*** **	**0.226** **	**0.141*** **
	(0.039)	**(0.039)**	**(0.014)**	**(0.097)**	**(0.085)**	**(0.045)**	**(0.109)**	**(0.098)**	**(0.050)**
^Imp Comp (ikt)	**0.068*** **	**0.090*** **	**-0.021*** **	**0.294** **	**0.296** **	**-0.002**	**0.502*** **	**0.585*** **	**-0.083**
	(0.014)	**(0.014)**	**(0.005)**	**(0.131)**	**(0.118)**	**(0.042)**	**(0.185)**	**(0.166)**	**(0.059)**
ln N Firms (ikt)	-0.321***	-0.248***	-0.073***	-0.257***	-0.185***	-0.072**	-0.292***	-0.196***	-0.097***
	(0.029)	(0.027)	(0.012)	(0.062)	(0.054)	(0.029)	(0.067)	(0.061)	(0.032)
Avg ln N Firms (kt)	0.327***	0.334***	-0.007	0.061	0.030	0.031			
	(0.046)	(0.046)	(0.019)	(0.127)	(0.123)	(0.052)			
Avg ln Employment (kt)	-0.461***	-0.458***	-0.003	0.054	0.021	0.033			
	(0.054)	(0.055)	(0.027)	(0.128)	(0.125)	(0.052)			
N	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777
R2	0.820	0.852	0.485	0.869	0.897	0.635	0.856	0.887	0.649
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sector FE	N	N	N	Y	Y	Y	N	N	N
Sector*Year FE	N	N	N	N	N	N	Y	Y	Y

Table 6. Additional Results

This table provides additional evidence on the impact of export demand and import competition on aggregate productivity at the country-sector-year level, based on Columns 1-3 and 7-9 in Table 5. Panel A weights observations at the country-sector level by the initial share of a sector in manufacturing employment. Panel B weights observations at the country-year level by the share of manufacturing in total employment. Panel C distinguishes between import competition from China vs. Rest Of the World. Panels D-E control for skill and mark-up dispersion across firms with the 90th-10th inter-percentile ratio in firm-level wages and price-to-cost margins. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%, 10%.

Dep Variable:	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Country-Sector Weights: Initial Share of Manuf Employment, L (ikt=0) / L^M (it=0)						
^Exp Dem (ikt)	0.427***	0.360***	0.067***	0.467***	0.359***	0.108***
	(0.039)	(0.036)	(0.011)	(0.102)	(0.090)	(0.039)
^Imp Comp (ikt)	0.075***	0.092***	-0.017***	0.498***	0.494***	0.004
	(0.015)	(0.014)	(0.005)	(0.151)	(0.141)	(0.043)
Panel B. Country-Year Weights: Manufacturing Share of Total Employment, L^M (it) / L (it)						
^Exp Dem (ikt)	0.385***	0.288***	0.097***	0.436***	0.267***	0.168***
	(0.037)	(0.036)	(0.013)	(0.112)	(0.101)	(0.052)
^Imp Comp (ikt)	0.069***	0.091***	-0.022***	0.703***	0.811***	-0.108*
	(0.014)	(0.014)	(0.005)	(0.193)	(0.175)	(0.063)
Panel C. Import Competition from China vs. ROW						
^Exp Dem (ikt)	0.371***	0.290***	0.082***	0.337***	0.200**	0.137***
	(0.038)	(0.038)	(0.013)	(0.104)	(0.093)	(0.047)
^Imp Comp ROW (ikt)	0.082***	0.086***	-0.004	0.398**	0.484***	-0.086
	(0.015)	(0.015)	(0.006)	(0.182)	(0.163)	(0.067)
^Imp Comp China (ikt)	-0.015	0.005	-0.019***	0.136**	0.141***	-0.005
	(0.014)	(0.014)	(0.004)	(0.058)	(0.051)	(0.023)
Panel D. Skill Dispersion across Firms						
^Exp Dem (ikt)	0.394***	0.291***	0.103***	0.364***	0.224**	0.140***
	(0.039)	(0.038)	(0.014)	(0.109)	(0.099)	(0.050)
^Imp Comp (ikt)	0.066***	0.088***	-0.022***	0.501***	0.584***	-0.083
	(0.014)	(0.014)	(0.005)	(0.184)	(0.165)	(0.059)
90-10 Wage Ratio (ikt)	-0.001**	-0.001**	-0.000	-0.001**	-0.001*	-0.000***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Panel E. Mark-Up Dispersion across Firms						
^Exp Dem (ikt)	0.397***	0.294***	0.103***	0.367***	0.226**	0.141***
	(0.039)	(0.039)	(0.014)	(0.109)	(0.098)	(0.050)
^Imp Comp (ikt)	0.068***	0.090***	-0.022***	0.509***	0.591***	-0.082
	(0.014)	(0.014)	(0.005)	(0.184)	(0.165)	(0.059)
90-10 PCM Ratio (ikt)	-0.000	-0.000	-0.000	-0.000*	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y
Sector*Year FE	N	N	N	Y	Y	Y

Table 7. Mechanisms: Selection and Innovation

This table examines the contribution of firm selection to the effects of export demand and import competition on aggregate productivity at the country-sector-year level. The outcome variable is indicated in the column heading and described in the text. All columns include country-year pair fixed effects and the full set of controls in Table 3. Columns 5-8 also include sector-year pair fixed effects. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%, 10%.

Panel A. Firm Selection

Dep Variable:	ln min Prod (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln min Prod (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
^Exp Dem (ikt)	**0.198*** **	**0.275*** **	**0.152*** **	**0.124*** **	**0.314*** **	**0.190*** **	**0.023**	**0.166*** **
	(0.040)	**(0.027)**	**(0.020)**	**(0.013)**	**(0.108)**	**(0.072)**	**(0.053)**	**(0.049)**
^Imp Comp (ikt)	**0.073*** **	**0.026*** **	**0.039*** **	**-0.013** **	**0.249**	**0.230* **	**0.324*** **	**-0.095**
	(0.015)	**(0.010)**	**(0.007)**	**(0.005)**	**(0.173)**	**(0.123)**	**(0.099)**	**(0.059)**
ln min Prod (ikt)		0.642***	0.733***	-0.091***		0.653***	0.676***	-0.023**
		(0.025)	(0.018)	(0.011)		(0.024)	(0.021)	(0.009)
N	2,750	2,750	2,750	2,750	2,750	2,750	2,750	2,750
R2	0.911	0.913	0.948	0.473	0.930	0.938	0.959	0.619
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y	Y	Y
Sector*Year FE	N	N	N	N	Y	Y	Y	Y

Panel B. Firm Selection & Innovation

Dep Variable:	ln R&D (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln R&D (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
^Exp Dem (ikt)	**0.103**	**0.282*** **	**0.154*** **	**0.129*** **	**0.370**	**0.237*** **	**0.055**	**0.182*** **
	(0.115)	**(0.027)**	**(0.019)**	**(0.012)**	**(0.448)**	**(0.083)**	**(0.057)**	**(0.052)**
^Imp Comp (ikt)	**0.164*** **	**0.016* **	**0.038*** **	**-0.022*** **	**-3.680*** **	**0.190**	**0.241** **	**-0.051**
	(0.046)	**(0.009)**	**(0.007)**	**(0.004)**	**(0.527)**	**(0.135)**	**(0.105)**	**(0.068)**
ln min Prod (ikt)		0.657***	0.736***	-0.079***		0.654***	0.676***	-0.022**
		(0.022)	(0.016)	(0.009)		(0.024)	(0.020)	(0.009)
ln R&D (ikt)		-0.000	-0.018***	0.017***		-0.018	-0.031***	0.012**
		(0.008)	(0.006)	(0.003)		(0.012)	(0.010)	(0.006)
N	2,777	2,750	2,750	2,750	2,777	2,750	2,750	2,750
R2	0.999	0.915	0.949	0.501	0.999	0.936	0.961	0.599
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y	Y	Y
Sector*Year FE	N	N	N	N	Y	Y	Y	Y

Table 8. Mechanisms: Imperfect Institutions and Market Frictions

This table examines the role of institutional efficiency in moderating the impact of export demand and import competition on aggregate productivity at the country-sector-year level. The outcome variable and the measure of institutional efficiency are indicated in the column heading and described in the text. All columns include country-year pair fixed effects and the full set of controls in Table 3. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%, 10%.

Institution Measure:	Rule of Law			(Inverse) Corruption			Labor Market Flexibility			Creditor Rights Protection			(Inverse) Product Market Regulation		
Dep. Variable:	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
^Exp Dem (ikt)	1.066***	0.862***	0.204***	0.850***	0.670***	0.180***	1.121***	0.763***	0.358***	0.718***	0.511***	0.207***	1.314***	1.047***	0.267***
	(0.126)	(0.111)	(0.037)	(0.096)	(0.085)	(0.031)	(0.261)	(0.238)	(0.063)	(0.158)	(0.147)	(0.040)	(0.172)	(0.155)	(0.045)
^Imp Comp (ikt)	-0.113**	-0.053	-0.060***	-0.063*	-0.013	-0.050***	-0.202**	-0.102	-0.100***	-0.108*	-0.063	-0.045***	-0.045	0.033	-0.078***
	(0.050)	(0.044)	(0.012)	(0.038)	(0.034)	(0.010)	(0.096)	(0.089)	(0.027)	(0.061)	(0.055)	(0.015)	(0.061)	(0.055)	(0.016)
^Exp Dem (ikt) x	**-0.476*** **	**-0.405*** **	**-0.070*** **	**-0.302*** **	**-0.252*** **	**-0.050*** **	**-0.218*** **	**-0.143** **	**-0.075*** **	**-0.048** **	**-0.033* **	**-0.015*** **	**-0.769*** **	**-0.636*** **	**-0.133*** **
Institution (it)	**(0.067)**	**(0.059)**	**(0.017)**	**(0.042)**	**(0.036)**	**(0.012)**	**(0.069)**	**(0.063)**	**(0.016)**	**(0.019)**	**(0.017)**	**(0.005)**	**(0.130)**	**(0.118)**	**(0.032)**
^Imp Comp (ikt) x	**0.136*** **	**0.106*** **	**0.030*** **	**0.095*** **	**0.074*** **	**0.021*** **	**0.083*** **	**0.060** **	**0.024*** **	**0.028*** **	**0.025*** **	**0.003**	**0.085* **	**0.039**	**0.046*** **
Institution (it)	**(0.031)**	**(0.028)**	**(0.006)**	**(0.020)**	**(0.018)**	**(0.004)**	**(0.027)**	**(0.026)**	**(0.008)**	**(0.009)**	**(0.008)**	**(0.002)**	**(0.046)**	**(0.043)**	**(0.013)**
N	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777
R2	0.792	0.835	0.459	0.797	0.839	0.460	0.747	0.802	0.447	0.811	0.848	0.463	0.825	0.858	0.398
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Appendix Table 1. Summary Statistics

This table provides summary statistics for the variation in aggregate productivity (CompNet) and trade activity (WIOD) across country-sector-year triplets in the 1998-2011 panel, as well as for the variation in institutional and market efficiency (World Justice Project, OECD, World Bank) across country-years in the 1998-2011 panel.

Panel A. Country-Sector-Year Level

	Years	# Sector-Years	Avg # Firms per Sector-Year	ln Aggregate Productivity		ln Average Productivity		Covariance Term		ln Exports	ln (Imports - Own-Sector Imp Inputs)
				Mean	St Dev	Mean	St Dev	Mean	St Dev		
AUSTRIA	2000-2011	178	68	4.29	0.53	4.23	0.52	0.06	0.09	8.06	6.67
BELGIUM	1998-2010	254	709	4.07	0.56	3.87	0.48	0.20	0.17	8.26	6.92
ESTONIA	1998-2011	157	218	1.96	0.58	1.63	0.60	0.33	0.22	4.93	3.70
FINLAND	1999-2011	233	573	4.06	0.56	3.88	0.52	0.18	0.20	7.10	5.65
FRANCE	1998-2009	231	3,559	4.03	0.47	3.85	0.44	0.19	0.15	9.14	8.05
GERMANY	1998-2011	274	721	4.50	0.40	4.39	0.38	0.11	0.09	9.91	8.62
HUNGARY	2003-2011	164	1,484	1.58	0.64	1.06	0.55	0.53	0.31	6.88	5.62
ITALY	2001-2011	218	4,356	3.53	0.43	3.25	0.44	0.28	0.09	9.17	7.75
LITHUANIA	2000-2011	179	263	1.86	0.61	1.38	0.58	0.48	0.23	5.01	4.17
POLAND	2005-2011	128	709	2.30	0.80	2.12	0.79	0.18	0.15	8.12	6.65
PORTUGAL	2006-2011	110	1,637	2.76	0.63	2.48	0.59	0.28	0.12	7.14	6.18
SLOVAKIA	2001-2011	182	109	2.11	0.63	1.97	0.57	0.14	0.20	6.60	5.26
SLOVENIA	1998-2011	232	216	2.30	0.58	2.20	0.54	0.10	0.17	6.06	4.74
SPAIN	1998-2011	271	3,192	3.46	0.44	3.15	0.38	0.31	0.15	8.39	7.42
Mean (across countries)		201	1,272	3.06	0.56	2.82	0.53	0.24	0.17	7.48	6.24
St Dev (across countries)		52	1,416	1.03	0.11	1.12	0.11	0.14	0.06	1.51	1.47

Appendix Table 1. Summary Statistics (cont,)

This table provides summary statistics for the variation in aggregate productivity (CompNet) and trade activity (WIOD) across country-sector-year triplets in the 1998-2011 panel, as well as for the variation in institutional and market efficiency (World Justice Project, OECD, World Bank) across country-years in the 1998-2011 panel.

Panel B. Country-Year Level

	Years	Rule of Law		Corruption		Labor Market Flexibility		Creditor Rights Protection		Product Market Regulation	
		Mean	St Dev	Mean	St Dev	Mean	St Dev	Mean	St Dev	Mean	St Dev
AUSTRIA	2000-2011	1.86	0.05	1.92	0.22	3.31	0.12	6.00	0.00	1.39	0.00
BELGIUM	1998-2010	1.29	0.06	1.37	0.08	3.18	0.04	5.00	0.00	1.18	0.00
ESTONIA	1998-2011	0.94	0.23	0.83	0.14	3.71	0.20	6.25	0.00	1.63	0.00
FINLAND	1999-2011	1.94	0.03	2.41	0.13	3.92	0.07	8.00	0.00	1.49	0.00
FRANCE	1998-2009	1.39	0.08	1.37	0.06	3.32	0.05	4.38	0.00	1.11	0.00
GERMANY	1998-2011	1.65	0.06	1.84	0.14	3.05	0.00	7.50	0.00	1.19	0.00
HUNGARY	2003-2011	0.85	0.08	0.48	0.15	3.60	0.00	7.00	0.00	1.03	0.00
ITALY	2001-2011	0.48	0.13	0.31	0.19	2.85	0.00	3.00	0.00	1.23	0.00
LITHUANIA	2000-2011	0.59	0.17	0.17	0.11			5.00	0.00		
POLAND	2005-2011	0.52	0.15	0.32	0.12	3.59	0.00	8.38	0.00	0.61	0.00
PORTUGAL	2006-2011	1.01	0.04	1.01	0.05	2.28	0.22	3.00	0.00	1.01	0.00
SLOVAKIA	2001-2011	0.47	0.11	0.28	0.16	3.28	0.10	8.00	0.00	1.11	0.00
SLOVENIA	1998-2011	0.98	0.10	0.94	0.15	3.15	0.02	4.50	0.00	1.11	0.00
SPAIN	1998-2011	1.19	0.09	1.19	0.16	3.25	0.03	6.00	0.00	1.07	0.00
Mean (across countries)		1.08	0.10	1.03	0.13	3.27	0.06	5.86	0.00	1.17	0.00
St Dev (across countries)		0.50	0.05	0.70	0.05	0.41	0.08	1.79	0.00	0.25	0.00

Appendix Table 2. Trade and Aggregate Productivity: OLS First Differences

This table examines the relationship between aggregate productivity and trade exposure at the country-sector-year level. The outcome variable is indicated in the column heading and described in the text. All left- and right-hand side variables are first differences over rolling 1-year, 3-year or 5-year overlapping periods. All columns include year fixed effects and the full set of controls in Table 3. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%, 10%.

	Δ = 1 year			Δ = 3 years			Δ = 5 years		
	Δ ln Agg Prod (ikt)	Δ ln Avg Prod (ikt)	Δ Cov Term (ikt)	Δ ln Agg Prod (ikt)	Δ ln Avg Prod (ikt)	Δ Cov Term (ikt)	Δ ln Agg Prod (ikt)	Δ ln Avg Prod (ikt)	Δ Cov Term (ikt)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Δ Exp Dem (ikt)	**0.116*** **	**0.034**	**0.082*** **	**0.142*** **	**0.053* **	**0.089*** **	**0.162*** **	**0.088*** **	**0.074*** **
	(0.028)	**(0.025)**	**(0.027)**	**(0.027)**	**(0.027)**	**(0.018)**	**(0.032)**	**(0.031)**	**(0.019)**
Δ Imp Comp (ikt)	**0.083*** **	**0.102*** **	**-0.019**	**0.062** **	**0.102*** **	**-0.040** **	**0.078*** **	**0.108*** **	**-0.030* **
	(0.021)	**(0.022)**	**(0.019)**	**(0.025)**	**(0.024)**	**(0.017)**	**(0.030)**	**(0.027)**	**(0.016)**
N	2,546	2,546	2,546	2,073	2,073	2,073	1,587	1,587	1,587
R2	0.114	0.115	0.022	0.101	0.117	0.044	0.096	0.094	0.035
Year FE, Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y

Appendix Table 3. Sensitivity Analysis

This table examines the stability of the impact of export demand and import competition on aggregate productivity at the country-sector-year level, based on Columns 1-3 and 7-9 in Table 5. Panels A-B consider only one dimension of trade exposure at a time. Panel C lags trade exposure by 1 year. Panel D measures import competition with the ratio of imports to domestic turnover. Panel E winsorizes productivity, trade, and foreign demand and supply instruments at the top and bottom 1 percentile. Standard errors clustered by sector-year in parentheses. ***, **, * significant at

Dep Variable:	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Only Export Demand						
^Exp Dem (ikt)	0.461***	0.350***	0.111***	0.417***	0.304***	0.114**
	(0.039)	(0.041)	(0.018)	(0.112)	(0.097)	(0.047)
Panel B. Only Import Competition						
^Imp Comp (ikt)	0.148***	0.149***	-0.001	0.730***	0.728***	0.001
	(0.013)	(0.015)	(0.005)	(0.150)	(0.142)	(0.050)
Panel C. Lagged Trade Exposure						
^Exp Dem (ikt-1)	0.395***	0.292***	0.103***	0.297***	0.179*	0.118**
	(0.041)	(0.041)	(0.014)	(0.102)	(0.092)	(0.049)
^Imp Comp (ikt-1)	0.069***	0.091***	-0.022***	0.500***	0.569***	-0.069
	(0.015)	(0.014)	(0.006)	(0.180)	(0.163)	(0.062)
Panel D. Import Competition Ratio						
^Exp Dem (ikt)	0.433***	0.329***	0.104***	0.465***	0.345***	0.121**
	(0.038)	(0.038)	(0.013)	(0.140)	(0.124)	(0.058)
^Imp Comp Ratio (ikt)	0.101***	0.144***	-0.043***	0.153***	0.181***	-0.028
	(0.020)	(0.020)	(0.010)	(0.053)	(0.047)	(0.024)
Panel E. Winsorizing Outliers						
^Exp Dem (ikt)	0.393***	0.301***	0.092***	0.206*	0.078	0.127*
	(0.039)	(0.039)	(0.014)	(0.120)	(0.122)	(0.067)
^Imp Comp (ikt)	0.073***	0.094***	-0.021***	0.637***	0.792***	-0.154*
	(0.014)	(0.014)	(0.006)	(0.245)	(0.236)	(0.087)
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y
Sector*Year FE	N	N	N	Y	Y	Y

Appendix Table 4. Imperfect Institutions and Market Frictions: Extensions

This table examines the stability of the role of institutional efficiency in moderating the impact of export demand and import competition on aggregate productivity at the country-sector-year level. Compared to Table 8, Panel A adds sector-year pair fixed effects, and Panel B considers different aspects of labor market flexibility. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%,

Panel A. Sector-Year Pair FE

Institution Measure:	Rule of Law			(Inverse) Corruption			Labor Market Flexibility			Creditor Rights Protection			(Inverse) Product Market Regulation		
	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
^Exp Dem (ikt)	1.902***	1.558***	0.343**	1.609***	1.243***	0.366**	-0.530*	-1.364***	0.835***	0.905*	0.142	0.762***	1.097***	0.910***	0.187***
	(0.429)	(0.359)	(0.152)	(0.411)	(0.327)	(0.156)	(0.319)	(0.296)	(0.287)	(0.516)	(0.340)	(0.293)	(0.222)	(0.220)	(0.065)
^Imp Comp (ikt)	-0.873**	-0.712**	-0.161	-0.859**	-0.655**	-0.204*	0.369**	0.491***	-0.122	0.179	0.416**	-0.237*	0.602***	0.701***	-0.099*
	(0.353)	(0.307)	(0.104)	(0.374)	(0.313)	(0.121)	(0.159)	(0.166)	(0.101)	(0.242)	(0.170)	(0.139)	(0.148)	(0.157)	(0.055)
^Exp Dem (ikt) x	**-0.754*****	**-0.653*****	**-0.101****	**-0.510*****	**-0.422*****	**-0.088****	**0.129**	**0.310*****	**-0.180*****	**-0.068**	**-0.009**	**-0.059****	**-0.683*****	**-0.602*****	**-0.082*****
Institution (it)	**(0.148)**	**(0.125)**	**(0.050)**	**(0.109)**	**(0.087)**	**(0.040)**	**(0.081)**	**(0.072)**	**(0.064)**	**(0.045)**	**(0.030)**	**(0.024)**	**(0.135)**	**(0.131)**	**(0.031)**
^Imp Comp (ikt) x	**0.177*****	**0.138*****	**0.039*****	**0.140*****	**0.107*****	**0.033*****	**-0.031**	**-0.090*****	**0.059*****	**0.039****	**0.017**	**0.022****	**0.085**	**0.066**	**0.018**
Institution (it)	**(0.048)**	**(0.042)**	**(0.011)**	**(0.038)**	**(0.031)**	**(0.010)**	**(0.029)**	**(0.028)**	**(0.022)**	**(0.019)**	**(0.013)**	**(0.010)**	**(0.061)**	**(0.060)**	**(0.017)**
N	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777
R2	0.727	0.808	0.549	0.731	0.821	0.487	0.896	0.907	0.431	0.840	0.904	0.086	0.856	0.876	0.642
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sector*Year FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Panel B. Sub-Components of Labor Market Flexibility

LMF Component:	Baseline: Regular Contracts (Individual & Collective)			Individual Regular Contracts			Collective Regular Contracts (Additional Provisions)			Temporary Employment Contracts			Synthetic Indicator (Regular & Temporary Contracts)		
	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)	ln Agg Prod (ikt)	ln Avg Prod (ikt)	Cov Term (ikt)
^Exp Dem (ikt)	1.121***	0.763***	0.358***	0.611***	0.482***	0.129***	0.376***	0.204**	0.172***	0.336	0.069	0.267***	0.276	0.028	0.248***
	(0.261)	(0.238)	(0.063)	(0.072)	(0.067)	(0.022)	(0.095)	(0.093)	(0.027)	(0.275)	(0.233)	(0.073)	(0.223)	(0.187)	(0.063)
^Imp Comp (ikt)	-0.202**	-0.102	-0.100***	-0.122***	-0.081***	-0.042***	-0.019	0.022	-0.040**	0.220***	0.270***	-0.050***	0.225***	0.275***	-0.050***
	(0.096)	(0.089)	(0.027)	(0.033)	(0.031)	(0.010)	(0.057)	(0.055)	(0.016)	(0.059)	(0.051)	(0.018)	(0.053)	(0.046)	(0.017)
^Exp Dem (ikt) x	**-0.218*****	**-0.143****	**-0.075*****	**-0.089*****	**-0.077*****	**-0.012****	**0.000**	**0.017**	**-0.017****	**0.014**	**0.060**	**-0.046****	**0.031**	**0.071**	**-0.041****
Institution (it)	**(0.069)**	**(0.063)**	**(0.016)**	**(0.016)**	**(0.015)**	**(0.005)**	**(0.017)**	**(0.017)**	**(0.007)**	**(0.071)**	**(0.059)**	**(0.019)**	**(0.056)**	**(0.047)**	**(0.016)**
^Imp Comp (ikt) x	**0.083*****	**0.060****	**0.024*****	**0.077*****	**0.068*****	**0.009****	**0.025***	**0.020**	**0.005**	**-0.040*****	**-0.048*****	**0.008***	**-0.042*****	**-0.050*****	**0.008***
Institution (it)	**(0.027)**	**(0.026)**	**(0.008)**	**(0.010)**	**(0.010)**	**(0.003)**	**(0.013)**	**(0.013)**	**(0.004)**	**(0.015)**	**(0.013)**	**(0.005)**	**(0.013)**	**(0.012)**	**(0.005)**
N	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777	2,777
R2	0.747	0.802	0.447	0.758	0.809	0.463	0.752	0.805	0.455	0.748	0.802	0.456	0.748	0.802	0.457
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Appendix Table 5. Trade and MRPK, MRPL, TFPR, Markup Dispersion

This table examines the impact of export demand and import competition on productivity and mark-up dispersion across firms at the country-sector-year level. The outcome variable is the standard deviation of the marginal revenue product of capital, the standard deviation of the marginal revenue product of labor, the standard deviation of revenue-based total factor productivity, or the 90th-10th interpercentile range of the price-cost mark-up as indicated in the column heading. All columns include country-year pair fixed effects and the full set of controls in Table 3. Columns 5-8 also include sector-year pair fixed effects. Standard errors clustered by sector-year in parentheses. ***, **, * significant at 1%, 5%, 10%.

Dep Variable:	MRPK St Dev	MRPL St Dev	TFPR St Dev	PCM p90 / p10	MRPK St Dev	MRPL St Dev	TFPR St Dev	PCM p90 / p10
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
^Exp Dem (ikt)	**-0.203*** **	**0.272*** **	**0.297*** **	**0.407*** **	**0.425*** **	**0.059**	**0.125**	**-0.738**
	(0.069)	**(0.038)**	**(0.035)**	**(0.138)**	**(0.145)**	**(0.082)**	**(0.155)**	**(0.527)**
^Imp Comp (ikt)	**0.193*** **	**0.095*** **	**0.059*** **	**-0.031**	**0.408* **	**0.483*** **	**0.981*** **	**2.077*** **
	(0.026)	**(0.012)**	**(0.013)**	**(0.050)**	**(0.229)**	**(0.131)**	**(0.248)**	**(0.707)**
N	2,777	2,777	2,382	2,775	2,777	2,777	2,382	2,775
R2	0.552	0.810	0.784	0.661	0.703	0.872	0.792	0.731
Ctry*Year FE, Controls	Y	Y	Y	Y	Y	Y	Y	Y
Sector*Year FE	N	N	N	N	Y	Y	Y	Y

Figure 1. Numerical Simulation: Welfare and Measured Aggregate Productivity

This figure illustrates the relationship between aggregate welfare, measured aggregate productivity, and the misallocation parameters in numerical model simulations. In each figure, the productivity-distortion correlation $\rho(\varphi,\eta)$ varies along the x-axis and the standard deviation of distortions σ_η varies along the y-axis. Figures A, B, C and D plot welfare, aggregate productivity, average productivity and the productivity-size covariance on the z-axis. All other parameter values are described in the text.

Figure 1A. Welfare

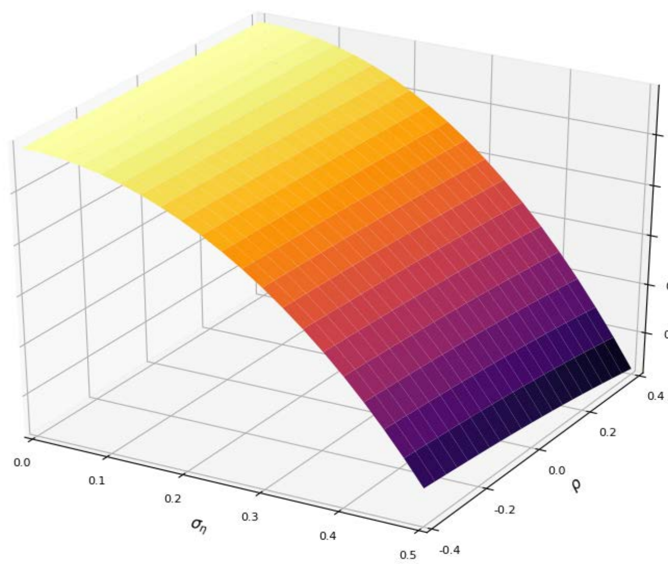


Figure 1B. (log) Aggregate Productivity

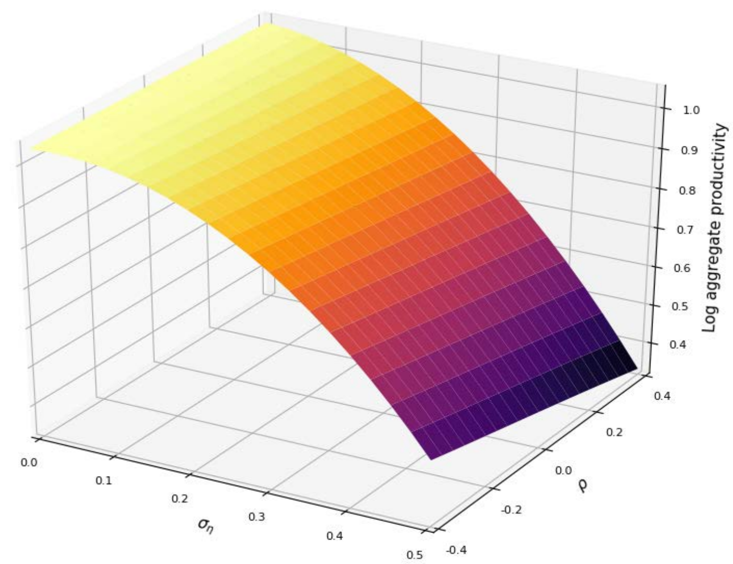


Figure 1C. (log) Average Productivity

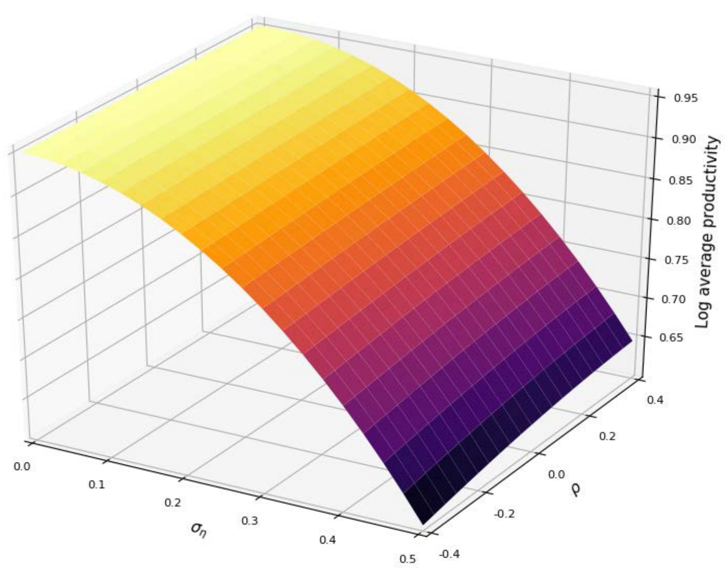


Figure 1D. (log) Productivity-Size Covariance

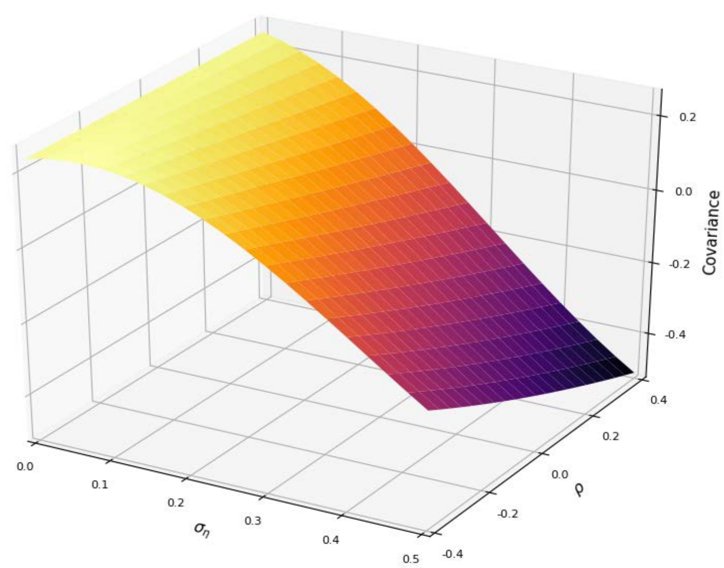
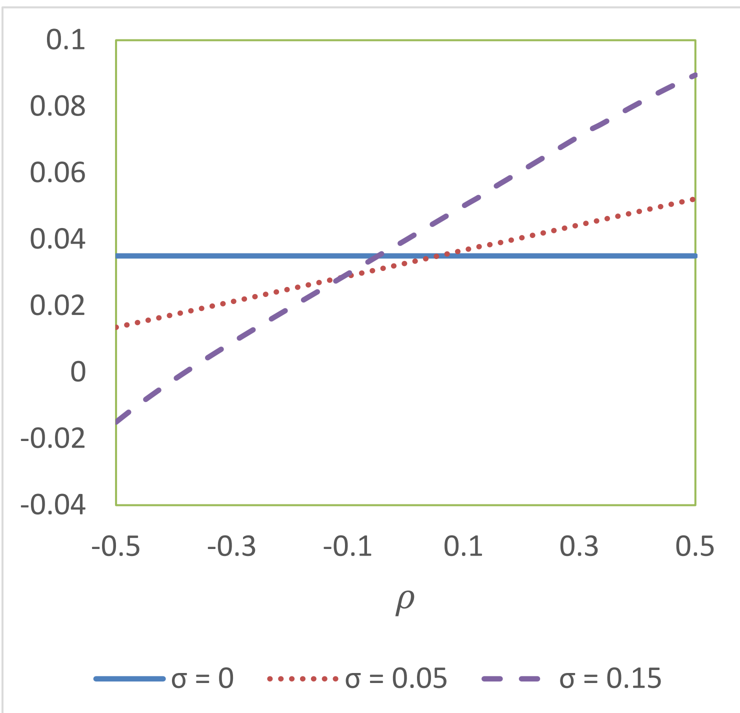


Figure 2. Numerical Simulation: Trade Liberalization

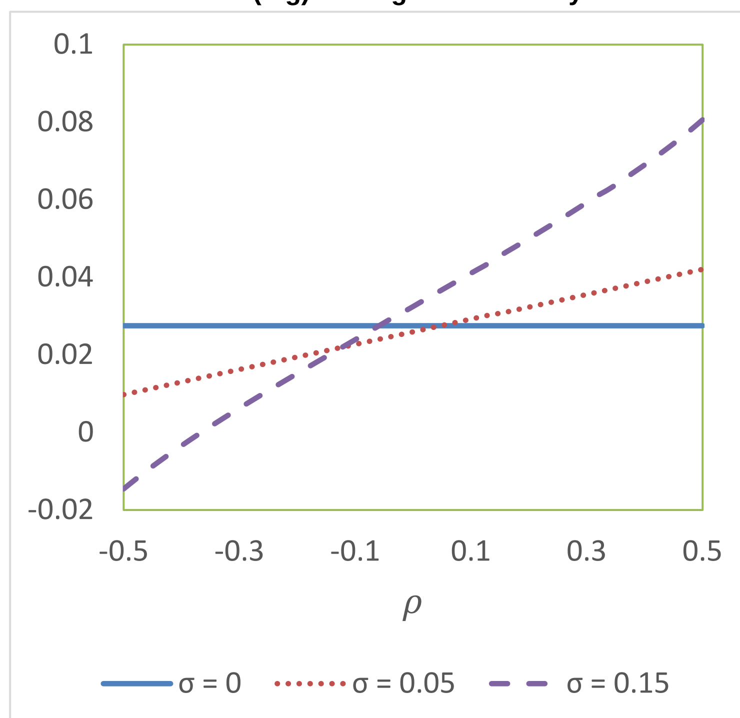
This figure displays numerical simulations for the productivity impact of reducing by 20% bilateral trade costs (Figure A) or unilateral export or import costs (Figure B-C). Each line shows how the predicted change in aggregate productivity, average productivity and the productivity-size covariance on the y-axis varies with the productivity-distortion correlation ρ(φ,η) on the x-axis. Different lines correspond to the case of no misallocation (standard deviation of distortions σ_η=0) and two cases of misallocation (σ_η={0.05,0.15}). All other parameter values are described in the text.

Figure 2A. Bilateral Trade Liberalization

(log) Aggregate Productivity



(log) Average Productivity



Productivity-Size Covariance

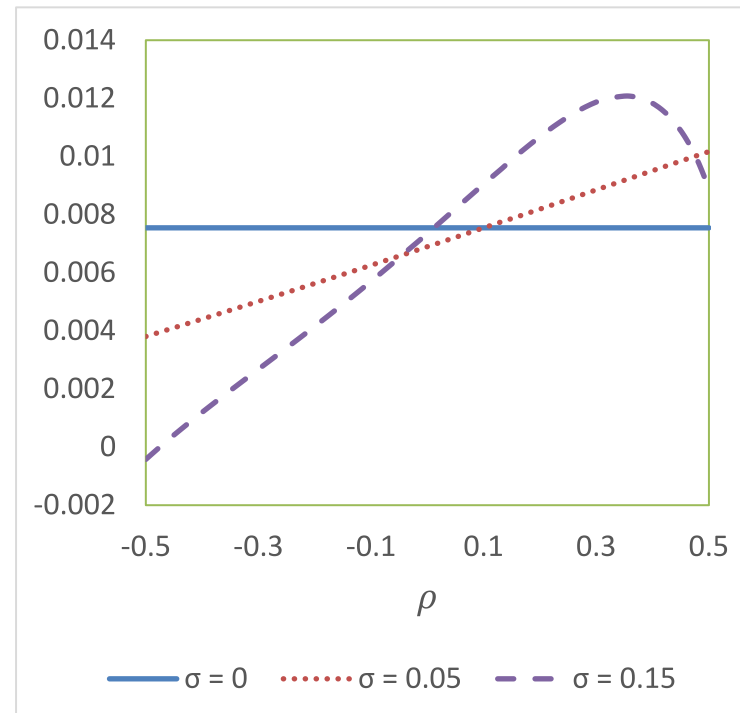


Figure 2. Numerical Simulation: Trade Liberalization (cont.)

This figure displays numerical simulations for the productivity impact of reducing by 20% bilateral trade costs (Figure A) or unilateral export or import costs (Figure B-C). Each line shows how the predicted change in aggregate productivity, average productivity and the productivity-size covariance on the y-axis varies with the productivity-distortion correlation $\rho(\varphi,\eta)$ on the x-axis. Different lines correspond to the case of no misallocation (standard deviation of distortions σ_η=0) and two cases of misallocation (σ_η={0.05,0.15}). All other parameter values are described in the text.

Figure 2B. Unilateral Export Liberalization

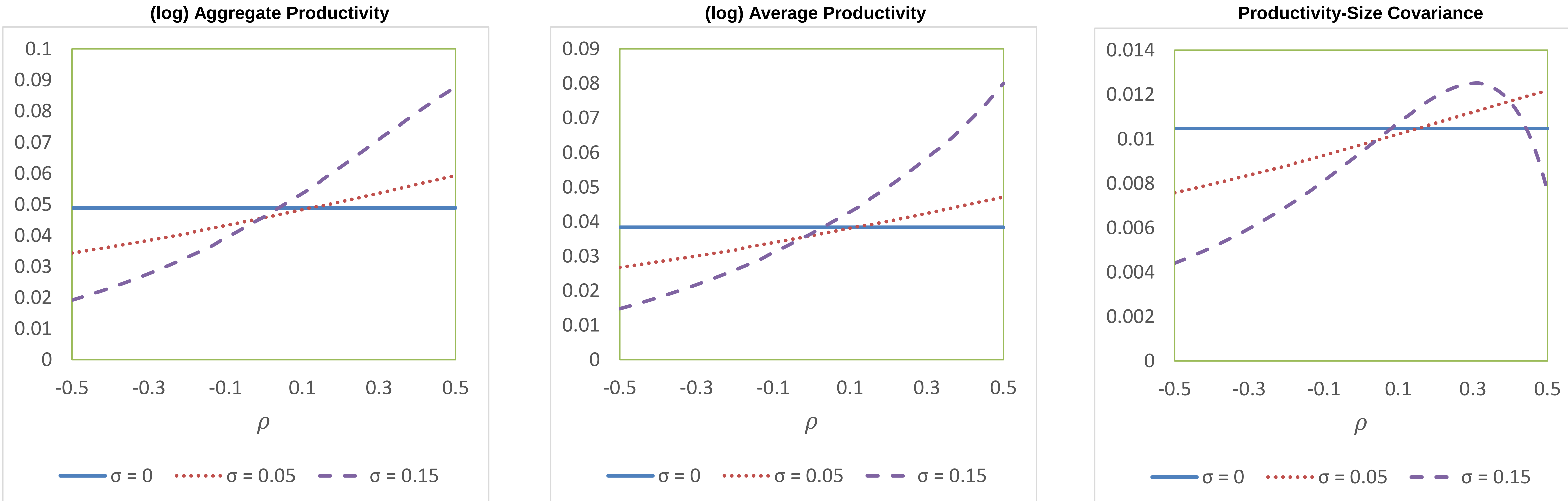


Figure 2C. Unilateral Import Liberalization

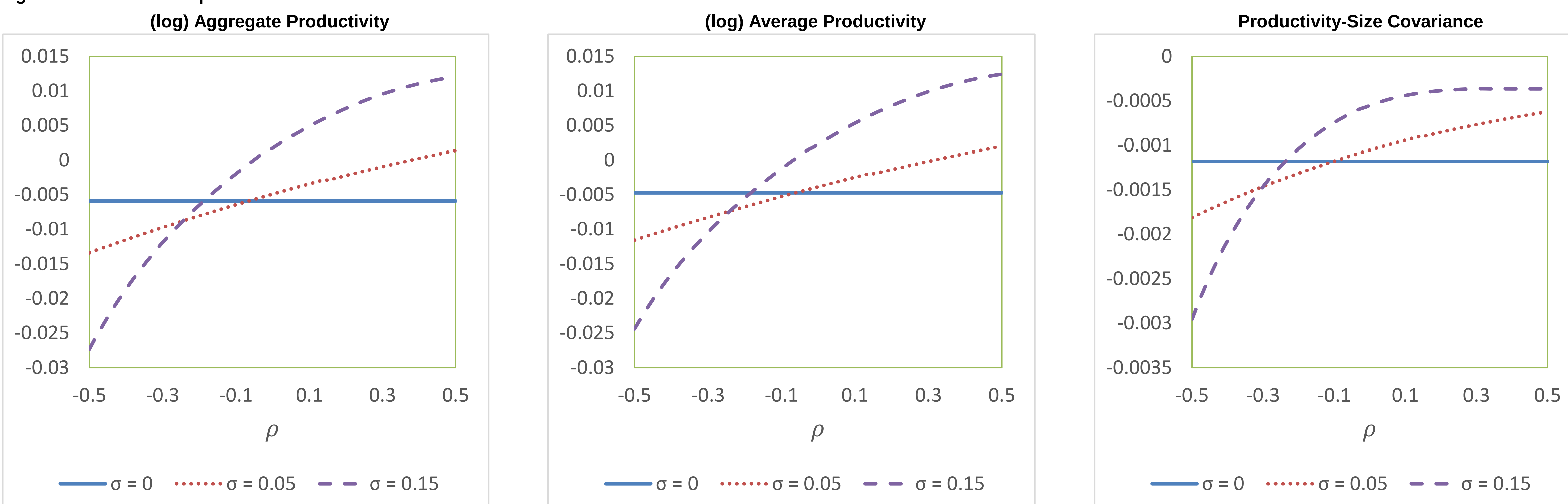


Figure 3. Sources of Productivity Growth

This figure displays the variation in the 3-year growth rate of aggregate productivity across countries in the panel. Each bar averages overlapping 3-year growth rates across sectors and years within a country. Figures A and B focus on the pre- and post-crisis periods of 2003-2007 and 2008-2011

Figure 3A. Growth 2003-2007

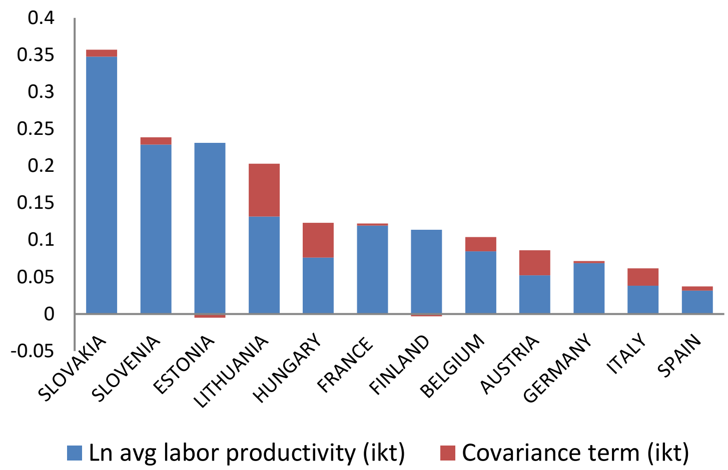


Figure 3B. Growth 2008-2011

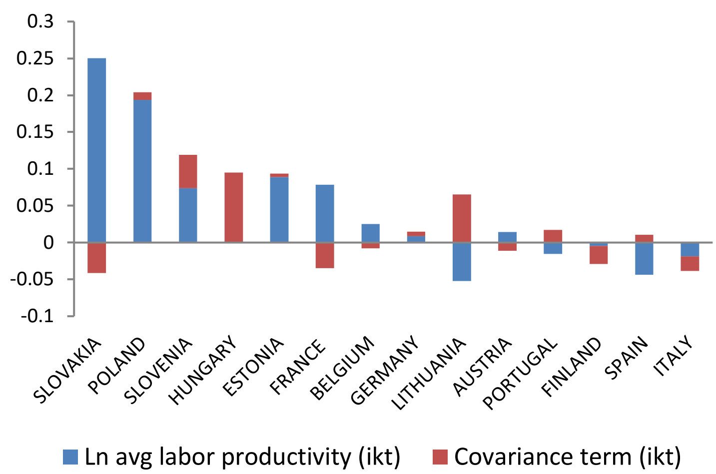


Figure 4. Trade Exposure Over Time

This figure displays the evolution of export and import activity in the panel. Each point represents an average value across countries and sectors in a given year. Each trade flow series is normalized to 1 in year 2000. Figure A covers all countries, while Figures B and C distinguish between EU-15 countries and new EU member states.

Figure 4A. All Countries

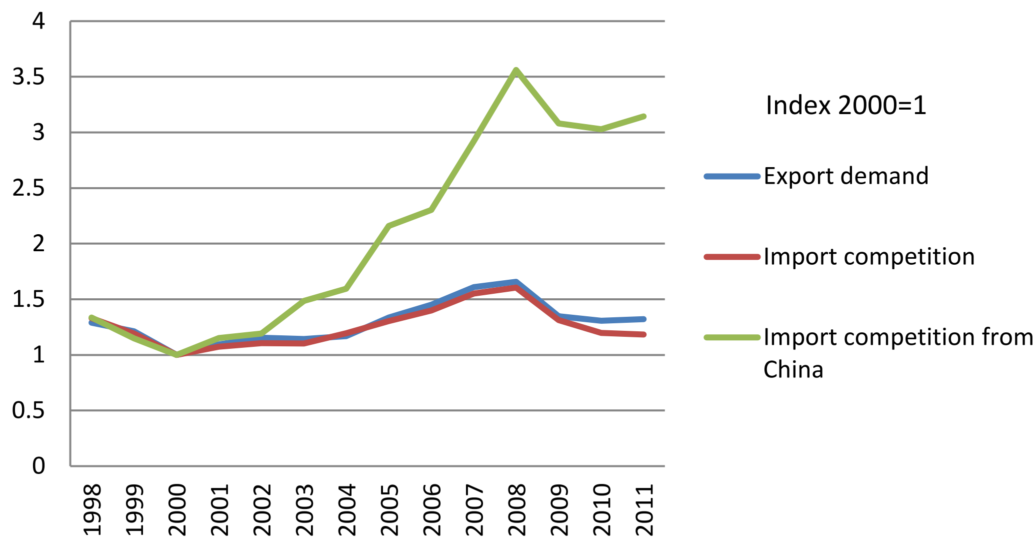


Figure 4B. New Member States

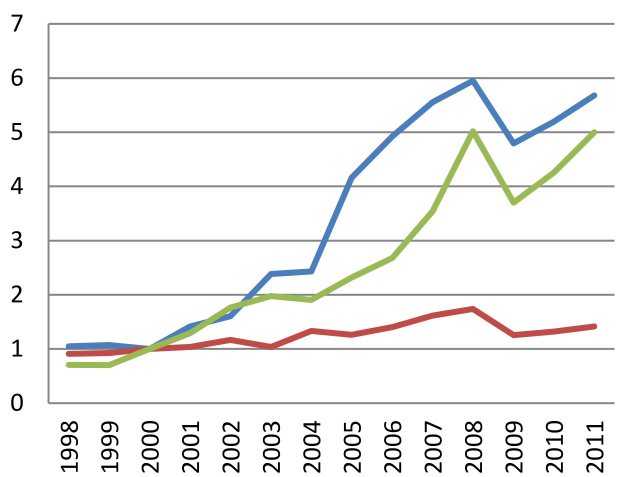


Figure 4C. EU-15 Countries

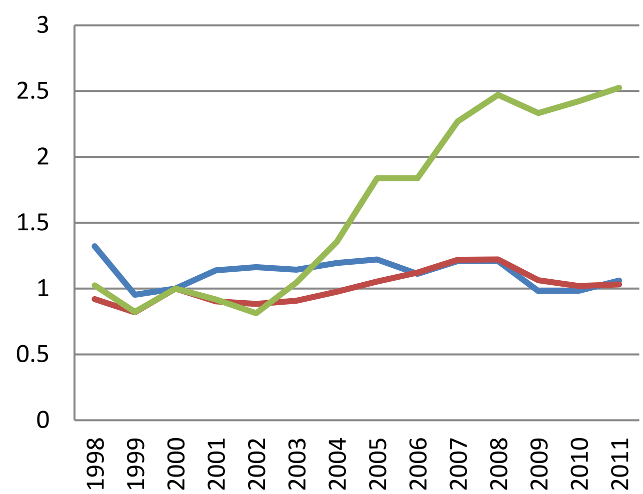
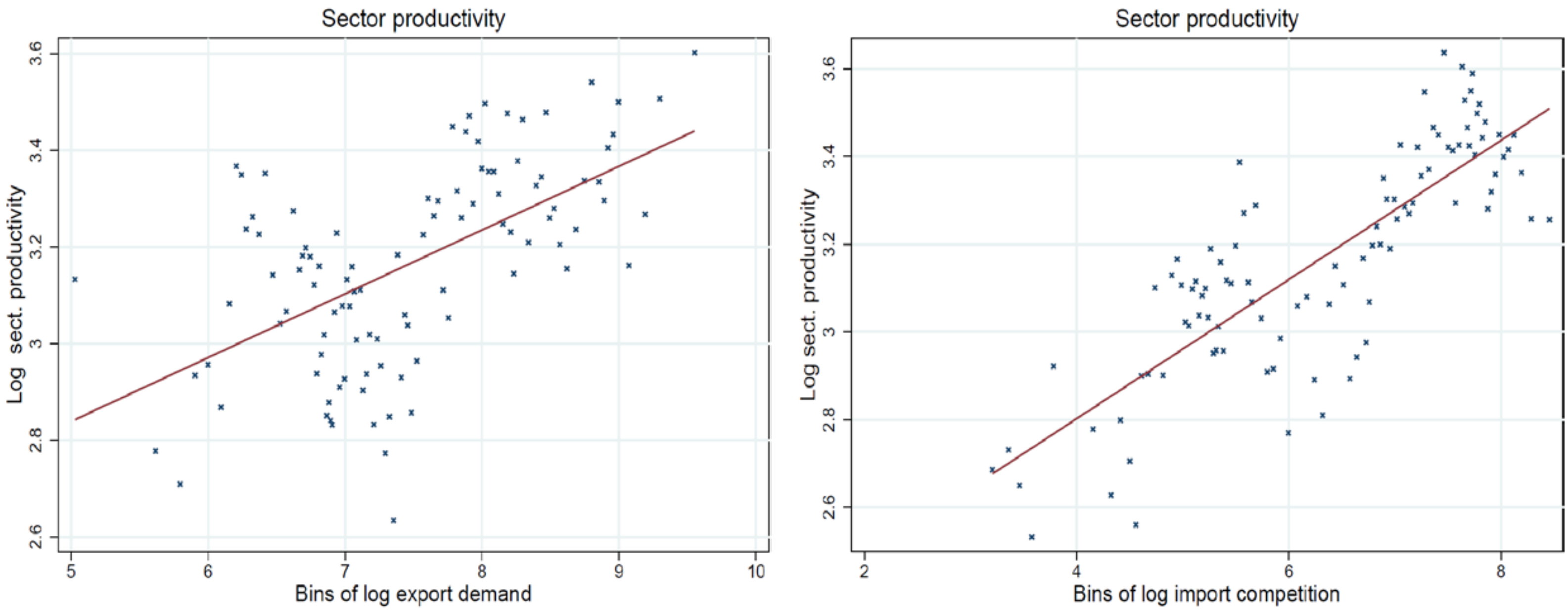


Figure 5. Trade Exposure and Aggregate Productivity

These bin scatters display the raw correlation of aggregate productivity with export and import activity across 100 bins in the panel. Each point represents average values across country-sector-year triplets within a percentile bin, after demeaning by country-year fixed



Trade, Productivity and (Mis)allocation: Appendix

Antoine Berthou*
Banque de France and *CEPII*

John Jong-Hyun Chung
Auburn

Kalina Manova
UCL and *CEPR*

Charlotte Sandoz
IMF

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Abstract

This Appendix complements Section 2 Theoretical Framework in the main paper. It provides a more detailed exposition of the model and formal proofs for all lemmas and propositions, but moves quickly over standard theoretical features discussed in the main paper.

Appendix Section 1 corresponds to Sections 2.1 and 2.2 in the paper. It introduces three model set-ups (efficient allocation and flexible wages, efficient allocation and fixed wages, and resource misallocation), derives firms' optimal behavior, describes the general equilibrium, and proves Lemma 1. Appendix Section 2 corresponds to Section 2.3 in the paper. It develops a mapping between theoretical concepts and empirical measures of productivity and welfare, and proves Lemma 2. Appendix Section 3 corresponds to Section 2.4 in the paper. It examines the impact of trade liberalization, and proves Propositions 1-3.

*Antoine Berthou: antoine.berthou@banque-france.fr. John Jong-Hyun Chung: jhc0054@auburn.edu. Kalina Manova (corresponding author): k.manova@ucl.ac.uk. Charlote Sandoz Dit Bragard: charl.sandoz@gmail.com.

1 Theoretical Framework: Three Model Set-ups

This section characterizes firm behavior and the general equilibrium in three versions of a heterogeneous-firm trade model with two countries.

The first subsection considers a single-sector model with optimal resource allocation, in which trade balance holds at the equilibrium and wages adjust in response to trade shocks. This set-up has been analyzed by Melitz (2003), Arkolakis et al (2012), and Demidova and Rodriguez-Clare (2013), among others.

The second subsection examines a two-sector model with optimal resource allocation, in which one sector produces a freely-traded, constant-returns-to-scale homogeneous good that fixes the wage.[1] This environment has been studied by Chaney (2008) and Demidova (2008).

The third subsection presents a model with resource misallocation, where firm-specific "wedges" lead firms to deviate from the socially optimal levels of production and exporting. This approach to modeling misallocation in the macro literature follows Hsieh and Klenow (2009) and Bartelsman et al (2013).

1.1 Efficient allocation and flexible wages

1.1.1 Set up and firm behavior

Country j has a mass L_j of consumers with CES preferences and utility

$$U_j = Q_j \equiv \left[\int_{z \in \Omega_j} q_j(z)^\alpha \, dz \right]^{1/\alpha} \tag{1.1}$$

where Ω_j is the set of varieties available in country j, $q_j(z)$ is the quantity of variety z consumed there, and $\sigma \equiv 1/(1-\alpha) > 1$ is the elasticity of substitution across varieties.

Country i has a mass of firms M_i that use labor to produce horizontally differentiated varieties. Entrepreneurs have to pay a sunk cost $w_i f_i^E$ to draw productivity $\varphi > 0$ from the Pareto distribution:

$$G_i(\varphi) = 1 - \left(\frac{\varphi_i^m}{\varphi} \right)^\theta, \quad \theta > \sigma - 1, \quad \varphi_i^m > 0. \tag{1.2}$$

A firm in country i with productivity φ needs to use $l_{ij}(q;\varphi)$ units of domestic labor to produce and deliver q units to market j, where

$$l_{ij}(q;\varphi) = f_{ij} + \frac{\tau_{ij} q}{\varphi}. \tag{1.3}$$

Here, $f_{ij} > 0$ represents the fixed overhead cost associated with sales to market j in units of labor, and $\tau_{ij} \geq 1$ represents the iceberg cost associated with delivery from i to j, with the normalization $\tau_{ii} = 1$. Each consumer provides a unit of labor inelastically.

The market is characterized by monopolistic competition with free entry. Firms' profit maximization problem can be separately solved for each destination. Profits from sales to market j are

$$\pi_{ij}(\varphi) = \max_{p,q} \, pq - w_i l_{ij}(q;\varphi) \tag{1.4}$$

[1] Since mark-ups will be 0 in the homogenous-good sector and positive in the differentiated-good sector, there is in principle a sub-optimal allocation of market shares across sectors. We abstract away from this dimension of misallocation to focus on distortions in the allocation of productive resources across heterogeneous firms in the differentiated sector.

where $q_j(z) = E_j P_j^{\sigma-1} p_j(z)^{-\sigma}$ is demand by country j consumers, E_j is aggregate expenditure in country j, $P_j \equiv \left[\int_{z\in\Omega_i} p_i(z)^{1-\sigma}\, dz\right]^{1/(1-\sigma)}$ is the consumer price index in country j, and w_i is the wage rate in country i. Firms' profit-maximizing quantity, price, revenues, costs and profits are then:

$$\begin{aligned}
q_{ij}(\varphi) &= E_j P_j^{\sigma-1}\left(\frac{\alpha\varphi}{w_i\tau_{ij}}\right)^{\sigma},\\
p_{ij}(\varphi) &= \frac{w_i\tau_{ij}}{\alpha\varphi},\\
r_{ij}(\varphi) &\equiv p_{ij}(\varphi)q_{ij}(\varphi) = E_j P_j^{\sigma-1}\left(\frac{\alpha\varphi}{w_i\tau_{ij}}\right)^{\sigma-1},\\
c_{ij}(\varphi) &\equiv w_i l_{ij}(q_{ij}(\varphi);\varphi) = \alpha r_{ij}(\varphi) + w_i f_{ij},\\
\pi_{ij}(\varphi) &= \frac{r_{ij}(\varphi)}{\sigma} - w_i f_{ij}.
\end{aligned}$$

Since profits are monotonically increasing in productivity, firms in country i sell in market j only if their productivity exceeds a certain threshold but not if $\pi_{ij}(\varphi) < 0$.

1.1.2 Equilibrium

Define the equilibrium as the set of cutoff productivity levels $\{\varphi_{ij}^*\}$, mass of firms $\{M_i\}$, wages $\{w_i\}$, price indices $\{P_i\}$, and expenditures $\{E_i\}$ that satisfy a system of equilibirum conditions for the zero-profit productivity cut-off, labor market clearing, free entry, price index, and income-expenditure balance.

The zero profit condition states that a firm with productivity φ in country i serves market j if and only if $\varphi \geq \varphi_{ij}^*$, where $\pi_{ij}(\varphi_{ij}^*) = 0$. This condition implies that:

$$\varphi_{ij}^* = \left(\frac{\sigma w_i f_{ij}}{E_j}\right)^{\frac{1}{\sigma-1}}\left(\frac{w_i\tau_{ij}}{\alpha P_j}\right). \tag{1.5}$$

The free entry condition requires that ex ante expected profits from entry equal the cost of entry, that is $\sum_j \mathbf{E}_i\left[\pi_{ij}(\varphi)\mathbf{I}(\varphi \geq \varphi_{ij}^*)\right] = w_i f_i^E$, where $\mathbf{E}_i[\cdot]$ is the expectation operator and $\mathbf{I}(\cdot)$ is the indicator function. Under Pareto distributed productivity, this condition can be expressed as:

$$f_i^E = \frac{\sigma-1}{\theta-(\sigma-1)}(\varphi_i^m)^{\theta}\sum_j f_{ij}(\varphi_{ij}^*)^{-\theta}. \tag{1.6}$$

Labor market clearing requires that total labor supplied L_i equal total labor employed in entry and production, $M_i f_i^E + M_i\left(\sum_j \mathbf{E}_i\left[l_{ij}(\varphi)\mathbf{I}(\varphi \geq \varphi_{ij}^*)\right]\right)$. Under Pareto, this condition simplifies to:

$$L_i = \frac{\sigma\theta}{\theta-(\sigma-1)}M_i(\varphi_i^m)^{\theta}\sum_j f_{ij}(\varphi_{ij}^*)^{-\theta} = \frac{\sigma\theta}{\sigma-1}M_i f_i^E, \tag{1.7}$$

where the second equality holds under the free entry condition (1.6). In particular, the mass of entrants in each country is invariant to trade costs:

$$M_i = \left(\frac{\sigma-1}{\sigma\theta}\right)\frac{L_i}{f_i^E}. \tag{1.8}$$

Since all firms with productivity φ charge the same price to a given destination, the consumer price index can be expressed in terms of $p_{ij}(\varphi)$ rather than $p_j(z)$. That is, $P_j^{1-\sigma} = \sum_i M_i \mathbf{E}\left[p_{ij}(\varphi)^{1-\sigma}\mathbf{I}(\varphi \geq \varphi_{ij}^*)\right]$. Under Pareto, this becomes:

$$P_j^{1-\sigma} = \frac{\theta}{\theta - (\sigma - 1)} \sum_i M_i \left(\frac{w_i \tau_{ij}}{\alpha}\right)^{1-\sigma} (\varphi_{ij}^*)^{\sigma-1-\theta} (\varphi_i^m)^\theta. \tag{1.9}$$

Finally, the income-expenditure balance requires that aggregate consumer expenditure equal aggregate earnings in each country:

$$E_j = P_j Q_j = w_j L_j. \tag{1.10}$$

Note that this condition implies balanced international trade. To see this, let X_{ij} denote aggregate sales from i to j. Then $X_{ij} = \frac{\sigma\theta}{\theta-(\sigma-1)} M_i w_i f_{ij} \left(\frac{\varphi_i^m}{\varphi_{ij}^*}\right)^\theta$, so that $\sum_j X_{ij} = \frac{\sigma\theta}{\theta-(\sigma-1)} M_i w_i (\varphi_i^m)^\theta \sum_j f_{ij} (\varphi_{ij}^*)^{-\theta} = w_i L_i = E_i$, where the second equality follows from (1.7) and the last equality follows from (1.10). Since aggregate expenditure satisfies $E_j = \sum_i X_{ij}$, trade balance will hold for each country k:

$$\sum_j X_{kj} = \sum_i X_{ik}. \tag{1.11}$$

The model does not guarantee $\varphi_{ii}^* \leq \varphi_{ij}^*$ for all possible parameters. To be consistent with the empirical evidence of selection into exporting, we restrict the parameter space so that $\varphi_{ii}^* \leq \varphi_{ij}^*$ holds. This requires fixed and variable export cost to be sufficiently high.

1.1.3 Welfare

Define welfare as real consumption per capita:

$$W_i \equiv \frac{Q_i}{L_i} = \frac{E_i}{P_i L_i} = \frac{w_i}{P_i} = \alpha \left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}} \varphi_{ii}^*, \tag{1.12}$$

where the first equality follows from the CES aggregation $E_i = Q_i P_i$, the second equality follows from the income-expenditure balance (1.10), and the last equality follows from the zero-profit condition (1.5).

A direct implication of (1.12) will be that any trade cost shock that increases the domestic productivity cut-off φ_{ii}^* will improve aggregate welfare. Likewise, any trade shock that reduces the expenditure share on domestic varieties will increase welfare, as ACR (2012) have shown. Since trade balance holds within the single differentiated-good sector, this will occur both due to trade shocks that increase the share of exports in total domestic production and due to trade shocks that increase the share of imports in total domestic consumption.[2]

[2] Let λ_k denote country k's expenditure share on domestic goods, which under balanced trade is equal to the share of the domestic market in domestic firms' total sales:

$$\lambda_k = \frac{X_{kk}}{\sum_i X_{ik}} = \frac{X_{kk}}{\sum_j X_{kj}} = \frac{\sigma - 1}{\theta - (\sigma - 1)} \frac{f_{kk}}{f_k^E} \left(\frac{\varphi_k^m}{\varphi_{kk}^*}\right)^\theta.$$

Hence,

$$d \log W_k = -\frac{1}{\theta} d \log \lambda_k.$$

In other words, any foreign supply or demand shock and any trade cost shock that increases the export sales share (which, under the model assumptions, must also increase the import consumption share) will improve welfare.

1.2 Efficient allocation and fixed wages

In the single-sector model, a unilateral reduction in export costs has the same effects as a unilateral reduction in import costs due to the equilibrium condition (1.11) that trade be balanced in the differentiated-good sector. One way to allow for asymmetric effects is to relax the balanced trade condition by introducing multiple sectors.

We introduce an "outside" sector that produces freely traded homogeneous goods. A unilateral export liberalization in the differentiated sector can and will now have opposite effects to a unilateral import liberalization. Intuitively, when the home country export cost goes down, home exports more. This increases competition in the foreign country, discouraging entry by foreign firms and reducing foreign's exports to home. The resulting imbalance between home's imports and exports of differentited goods can be maintained as the foreign country can specialize in the outside sector.

1.2.1 Set up and firm behavior

Country j has a mass L_j of consumers with nested utility:

$$U_j = H_j^{1-\beta} Q_j^{\beta},$$

where H_j is the quantity of the homogeneous good consumed and Q_j is as in (1.1). A unit of labor produces w_i units of the homogeneous good in country i, which is freely traded and chosen as the numeraire. The labor market is competitive and labor is mobile across sectors, so the wage in country i is w_i. The aggregate price index is now $P_i = P_{iQ}^{\beta}$, where P_{iQ} is the differentiated-good sector price index.

The market for differentiated goods is characterized by monopolistic competition with production and trade technology as before. The firm's profit maximization problem therefore delivers the same first-best solution as above, adjusted for the share of aggregate expenditure βE_j and the price index P_{iQ} relevant for the differentiated sector:

$$\begin{aligned}
q_{ij}(\varphi) &= \beta E_j P_{jQ}^{\sigma-1} \left(\frac{\alpha\varphi}{w_i\tau_{ij}}\right)^{\sigma}, \\
p_{ij}(\varphi) &= \frac{w_i\tau_{ij}}{\alpha\varphi}, \\
r_{ij}(\varphi) &\equiv p_{ij}(\varphi)q_{ij}(\varphi) = \beta E_j \left(P_{jQ}\right)^{\sigma-1} \left(\frac{\alpha\varphi}{w_i\tau_{ij}}\right)^{\sigma-1}, \\
c_{ij}(\varphi) &\equiv w_i l_{ij}(q_{ij}(\varphi);\varphi) = \alpha r_{ij}(\varphi) + w_i f_{ij}, \\
\pi_{ij}(\varphi) &= \frac{r_{ij}(\varphi)}{\sigma} - w_i f_{ij}.
\end{aligned}$$

1.2.2 Equilibrium

The equilibrium cutoffs $\{\varphi_{ij}^*\}$, mass of firms $\{M_i\}$, price indices $\{P_i, P_{iQ}\}$, and aggregate expenditures $\{E_i\}$ are determined by the conditions above for zero cut-off profits (1.5), free entry (1.6), and income-expenditure balance (1.10), along with a modified expression for the price index:

$$P_{jQ}^{1-\sigma} = \frac{\theta}{\theta-(\sigma-1)} \sum_i M_i \left(\frac{w_i\tau_{ij}}{\alpha}\right)^{1-\sigma} (\varphi_{ij}^*)^{\sigma-1-\theta} (\varphi_i^m)^{\theta}, \tag{1.13}$$

Note that the earlier labor market clearing condition (1.7) no longer binds and is therefore excluded from the current equilibrium. In other words, the quantity of labor demanded by the differentiated goods sector (the right-hand side of (1.7)) is strictly less than the quantity of labor available, L_i. The residual labor is used in the production of the homogeneous good.

The equilibrium conditions here assume imperfect specialization. Under sufficiently strong asymmetry, one country may completely specialize in the differentiated goods sector. In that case the mass of firms in the other country will be zero, and the specialized country's cutoffs and mass of firms will be determined by the free entry condition and consumers' budget constraint.

1.2.3 Welfare

Aggregate welfare can be expressed as:

$$W_i \equiv \frac{U_i}{L_i} = (1-\beta)^{1-\beta}\beta^{\beta}\frac{w_i}{P_{iQ}^{\beta}} = ((1-\beta)w_i)^{1-\beta}\left(\alpha\beta\left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}}\varphi_{ii}^*\right)^{\beta}. \tag{1.14}$$

Thus φ_{ii}^* is still a sufficient statistic for welfare, and aggregate welfare increases with the domestic productivity cut-off. Unlike the case of the single-sector model above, however, trade balance no longer holds within the differentiated-good sector. As a result, trade shocks that increase the share of exports in total domestic production will increase welfare, but the same need not hold for the share of imports in domestic consumption.[3]

[3] The share of home sales in domestic firms' total sales is still given by:

$$\lambda_k^X \equiv \frac{X_{kk}}{\sum_j X_{kj}} = \frac{\sigma-1}{\theta-(\sigma-1)}\frac{f_{kk}}{f_k^E}\left(\frac{\varphi_k^m}{\varphi_{kk}}\right)^{\theta},$$

so that $d\log W_k = -\frac{\beta}{\theta}d\log\lambda_k^X$. However, the trade balance condition no longer holds within the differentiated sector, such that the share of domestic goods in total domestic consumption is $\lambda_k^M \equiv \frac{X_{kk}}{\sum_i X_{ik}} \neq \lambda_k^X$.

In the case of two countries, one can show that

$$\varphi_{11} = \left(\frac{a_{22}\tilde{f}_1^E - a_{12}\tilde{f}_2^E}{a_{11}a_{22}-a_{12}a_{21}}\right)^{-\frac{1}{\theta}}. \tag{1.15}$$

Therefore, a unilateral import liberalization in country 1 that reduces f_{21} or τ_{21} and thus increases a_{21} will decrease φ_{11} and depress welfare in country 1. On the other hand, a unilateral export liberalization in country 1 that increases a_{12} will raise φ_{11} and welfare in country 1, as expected.

This result can be understood as a delocation effect. In the two-country case, the mass of entrants in country 1 is:

$$M_1 = \frac{\tilde{a}_{22}\tilde{\varphi}_{11}^{\theta} - \tilde{a}_{12}\tilde{\varphi}_{22}^{\theta}}{\tilde{a}_{11}\tilde{a}_{22}-\tilde{a}_{12}\tilde{a}_{21}}.$$

A fall in import trade costs — which increases $\tilde{a}_{21}$, decreases $\tilde{\varphi}_{11}$, and increases $\tilde{\varphi}_{22}$ — will reduce M_1. This loss of domestic varieties outweighs the gain from foreign varieties and associated price changes, leading to a net decline in welfare.

More generally, one can show that:

$$\lambda_k^M = \frac{\sigma\theta}{\theta-(\sigma-1)}\frac{f_{kk}}{L_k}M_k\left(\frac{\varphi_k^m}{\varphi_{kk}}\right)^{\theta}.$$

Hence, any shock that simultaneously increases the import share in consumption λ_k^M and decreases the mass of domestic entrants M_k will necessarily decrease the domestic cutoff φ_{kk} and subsequently welfare.

1.3 Resource misallocation

We now introduce resource misallocation in the standard heterogeneous-firm trade model. We consider the case of an outside sector to allow unilateral export and import liberalizations to have asymmetric effects. The equilibrium of the single-sector alternative can be obtained by adjusting the conditions below analogously to the adjustments between Sections 1.1 and 1.2 above.

We introduce firm-specific "wedges" that generate deviations from the socially optimal resource allocation across firms. We refer to these wedges as subsidies, but they capture the net effect of all possible factors that cause a firm to deviate from the first-best levels of production and exporting. Consequently, some firms become larger than optimal while others remain smaller than optimal.

1.3.1 Set up

After paying a sunk entry cost of $w_i f_i^E$, each entrant receives two draws, productivity $\varphi > 0$ and production subsidy/tax $\eta > 0$, from a joint distribution $H_i(\varphi, \eta)$. For comparability with the no-misallocation models, we assume $\underline{\varphi}$ is Pareto distributed with scale parameter $\underline{\varphi}_i^m$ and shape parameter θ, which will imply that the observed distribution of firm sales follows Pareto.

Firms' production technology is still characterized by its productivity through (1.3). The subsidy η affects only the production cost conditional on the amount of labor used, so that the cost to the firm associated with manufacturing q units is:

$$c_{ij}(q; \varphi, \eta) = w_i \left(f_{ij} + \frac{\tau_{ij} q}{\eta \varphi} \right).$$

This differs from the pre-subsidy cost, i.e. the wage payments received by workers:

$$c'_{ij}(q; \varphi, \eta) = w_i \left(f_{ij} + \frac{\tau_{ij} q}{\varphi} \right).$$

The profits of a firm with productivity φ and subsidy η in destination market j are therefore:

$$\pi_{ij}(\varphi, \eta) = \max_{p,q} \quad pq - c_{ij}(q; \varphi, \eta). \tag{1.16}$$

Firms' profit-maximizing quantity, price, revenues, costs and profits are then:

$$\begin{aligned}
q_{ij}(\varphi, \eta) &= \beta E_j P_{jQ}^{\sigma-1} \left(\frac{\alpha \varphi \eta}{w_i \tau_{ij}} \right)^{\sigma}, \\
p_{ij}(\varphi, \eta) &= \frac{w_i \tau_{ij}}{\alpha \varphi \eta}, \\
r_{ij}(\varphi, \eta) &\equiv p_{ij}(\varphi, \eta) q_{ij}(\varphi, \eta) = \beta E_j P_{jQ}^{\sigma-1} \left(\frac{w_i \tau_{ij}}{\alpha} \right)^{1-\sigma} (\varphi \eta)^{\sigma-1}, \\
c_{ij}(\varphi, \eta) &\equiv c_{ij}(q_{ij}(\varphi, \eta); \varphi, \eta) = \alpha \eta r_{ij}(\varphi, \eta) + w_i f_{ij}, \\
c'_{ij}(\varphi, \eta) &\equiv c'_{ij}(q_{ij}(\varphi, \eta); \varphi, \eta) = \alpha r_{ij}(\varphi, \eta) + w_i f_{ij}, \\
\pi_{ij}(\varphi, \eta) &= \frac{r_{ij}(\varphi, \eta)}{\sigma} - w_i f_{ij}.
\end{aligned}$$

1.3.2 Equilibrium

Define the distorted productivity of a firm as $\underline{\varphi} \equiv \varphi\eta$. Note that firm profits depend on firm characteristics (φ, η) through and only through distorted productivity $\underline{\varphi}$. In addition, profits are monotonically increasing in $\underline{\varphi}$. This implies that there exists a unique $\underline{\varphi}^*_{ij}$ defined by $\pi_{ij}(\underline{\varphi}^*_{ij}) = 0$, such that all firms with $\underline{\varphi} > \underline{\varphi}^*_{ij}$ can profitably sell to market j:

$$\underline{\varphi}^*_{ij} = \left(\frac{\sigma w_i f_{ij}}{\beta E_j}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_i \tau_{ij}}{\alpha P_{jQ}}\right). \tag{1.17}$$

The free entry condition implies that ex ante expected profits equal the sunk cost of entry:

$$f_i^E = \frac{\sigma - 1}{\theta - (\sigma - 1)} \left(\underline{\varphi}_i^m\right)^\theta \sum_j f_{ij} \left(\underline{\varphi}^*_{ij}\right)^{-\theta}. \tag{1.18}$$

Note that (1.17) is equivalent to (1.5) and (1.18) is equivalent to (1.6), with productivity φ in the no-misallocation case replaced by distorted productivity $\underline{\varphi}$ in the misallocation case.

The consumer budget constraint, however, is substantially different. Assume that subsidies to firms producing in country i are covered by lump-sum taxation of consumers in i. Aggregate income in country i is then total labor income less the aggregate cost of all subsidies:

$$E_i = w_i L_i - T_i \tag{1.19}$$

where

$$T_i \equiv C_i' - C_i = \sum_j M_i w_i f_{ij}(\sigma - 1) \iint_{\varphi\eta \geq \underline{\varphi}^*_{ij}} (\eta - 1) \left(\frac{\varphi\eta}{\underline{\varphi}^*_{ij}}\right)^{\sigma-1} dH_i(\varphi, \eta). \tag{1.20}$$

The equilibrium cut-off profitability levels $\{\underline{\varphi}^*_{ij}\}$ and the mass of firms $\{M_i\}$ are characterized by equations (1.17), (1.18), and (1.19).

1.3.3 Welfare

The welfare of country i can be expressed as:

$$W_i = (1-\beta)^{1-\beta}\beta^\beta \frac{E_i}{P_i L_i} = (1-\beta)^{1-\beta}\beta^\beta \left(\frac{w_i}{P_i}\right) \chi_i = ((1-\beta) w_i)^{1-\beta} \left(\alpha\beta \left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}} \underline{\varphi}^*_{ii}\right)^\beta \chi_i^{\frac{\beta + (\sigma-1)}{\sigma-1}}, \tag{1.21}$$

where the share of disposable income available to consumers is:

$$\chi_i \equiv \frac{w_i L_i - T_i}{w_i L_i}.$$

From (1.20), the aggregate tax T_i and hence χ_i depend on the joint distribution of (φ, η), and cannot be determined from the marginal distribution of $\underline{\varphi}$ alone. The aggregate tax T_i may either increase or decrease in response to a rise in $\underline{\varphi}^*_{ii}$ even when $\underline{\varphi}$ follows Pareto, depending on the joint distribution of (φ, η). Moreover, a potential increase in T_i can be sufficiently high such that welfare can fall in response to a rise in $\underline{\varphi}^*_{ii}$. This stands in sharp contrast to the no-misallocation model.

1.4 Proof of Lemma 1

Equations (1.12), (1.14) and (1.21) imply that aggregate welfare is proportional to the productivity cut-off for domestic production in the absence of misallocation and to the profitability cut-off and the share of disposable income in the presence of misallocation:

$$W_i \propto \left\{ \begin{array}{ll} \left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{\beta}{\sigma-1}} (\varphi_{ii}^*)^{\beta} & \text{without misallocation} \\ \left(\frac{L_i}{\sigma f_{ii}}\right)^{\frac{\beta}{\sigma-1}} (\chi_i)^{\frac{\beta+\sigma-1}{\sigma-1}} (\underline{\varphi}_{ii}^*)^{\beta} & \text{with misallocation} \end{array} \right\}. \tag{1.22}$$

The proves Lemma 1 as stated in the paper:

Lemma 1 *Without misallocation, welfare increases with the domestic productivity cut-off,* $\frac{dW_i}{d\varphi_{ii}^*} > 0$*. With misallocation, welfare increases with the distorted domestic productivity cut-off (holding* χ_i *fixed),* $\frac{\partial W_i}{\partial \underline{\varphi}_{ii}^*} > 0$*, and with the share of disposable income in gross income (holding* $\underline{\varphi}_{ii}^*$ *fixed),* $\frac{\partial W_i}{\partial \chi_i} > 0$*.*

2 From Theory to Empirics

We now consider the relationship between the theoretical concepts of welfare, firm productivity, and aggregate productivity and their empirical counterparts that can be measured in the data. For the case of real value added per worker, we establish that the measured aggregate productivity of domestic firms is proportional to welfare in the absence of misallocation, but not in the presence of misallocation.

2.1 Theoretical and measured firm productivity

In Section 2.3 of the paper, we introduce real value added per worker $\Phi_i(\varphi)$ as the empirical counterpart to firm productivity in the model φ. Observed value added corresponds to total firm revenues from domestic sales and any exports, $r_i(\varphi) = \sum_j r_{ij}(\varphi)\mathbf{I}(\varphi \geq \varphi_{ij}^*)$. Observed employment represents the total amount of labor that a firm hires to produce for home and abroad, $l_i(\varphi) = \sum_j l_{ij}(\varphi)\mathbf{I}(\varphi \geq \varphi_{ij}^*)$. Denoting labor used towards fixed overhead and export costs as $f_i(\varphi) = \sum_j f_{ij}\mathbf{I}(\varphi \geq \varphi_{ij}^*)$ and normalizing by the consumer price index in the differentiated industry $P_{iQ} = P_i^{1/\beta}$, measured firm productivity is given by:

$$\Phi_i(\varphi) = \frac{r_i(\varphi)}{P_i^{1/\beta} l_i(\varphi)} = \frac{w_i}{\alpha P_i^{1/\beta}} \left[1 - \frac{f_i(\varphi)}{l_i(\varphi)}\right]. \tag{2.1}$$

In Section 2.3 of the paper, we claim that measured productivity $\Phi_i(\varphi)$ is monotonically increasing in theoretical productivity φ conditional on export status, i.e. $\Phi_i'(\varphi|\varphi < \varphi_{ij}^*) > 0$ and $\Phi_i'(\varphi|\varphi \geq \varphi_{ij}^*) > 0$. From equation (2.1), it is sufficient to show that $l_{ii}(\varphi)$ and $l_{ii}(\varphi) + l_{ij}(\varphi)$ are increasing in φ. The latter follows from the firm's maximization problem since $l_{ii}(\varphi) = f_{ii} + \beta E_i P_{iQ}^{\sigma-1} \left(\frac{\alpha}{w_i}\right)^{\sigma} \varphi^{\sigma-1}$ and $l_{ii}(\varphi) + l_{ij}(\varphi) = (f_{ii} + f_{ij}) + \beta \left(E_i P_{iQ}^{\sigma-1} + E_j P_{jQ}^{\sigma-1} \tau_{ij}^{1-\sigma}\right) \varphi^{\sigma-1}$, both of which are increasing in φ.

In the case of misallocation, there is an analogous relationship between theoretical and observed distorted productivity, $\underline{\varphi} = \varphi\eta$ and $\underline{\Phi}_i(\varphi, \eta)$. Now measured firm productivity is monotonically increasing in distorted productivity conditional on export status.

$$\underline{\Phi}_i(\varphi,\eta) = \frac{r_i(\varphi,\eta)}{P_i^{1/\beta} l_i(\varphi,\eta)} = \frac{w_i}{\alpha P_i^{1/\beta}\eta}\left[1 - \frac{f_i(\varphi,\eta)}{l_i(\varphi,\eta)}\right]. \tag{2.2}$$

2.2 Theoretical and measured aggregate productivity

In Section 2.3 of the paper, we define measured aggregate productivity in the differentiated-good sector:

$$\tilde{\Phi}_i = \begin{cases} \int_{\varphi_{ii}^*}^{\infty} \theta_i(\varphi)\Phi_i(\varphi)\,\frac{dG_i(\varphi)}{1-G_i(\varphi_{ii}^*)} & \text{without misallocation} \\ \int_{\underline{\varphi}_{ii}^*}^{\infty} \theta_i(\varphi,\eta)\Phi_i(\varphi,\eta)\,\frac{dG_i(\varphi,\eta)}{1-G_i(\underline{\varphi}_{ii}^*)} & \text{with misallocation} \end{cases}, \tag{2.3}$$

where $\theta_i(\varphi) = l_i(\varphi)/\left[\int_{\varphi_{ii}^*}^{\infty} l_i(\varphi)\,\frac{dG_i(\varphi)}{1-G_i(\varphi_{ii}^*)}\right]$ and $\theta_i(\varphi,\eta) = l_i(\varphi,\eta)/\left[\int_{\underline{\varphi}_{ii}^*}^{\infty} l_i(\varphi,\eta)\,\frac{dG_i(\varphi,\eta)}{1-G_i(\underline{\varphi}_{ii}^*)}\right]$ are a firm's share of aggregate employment.[4]

In Section 2.3, we claim that $\tilde{\Phi}_i$ can be expressed as:

$$\tilde{\Phi}_i = \begin{cases} \frac{\sigma\theta}{\sigma\theta-(\sigma-1)}\frac{w_i}{P_i^{1/\beta}} & \text{without misallocation} \\ \frac{\sigma\theta}{(\sigma-1)\theta\tilde{\eta}_i+\theta-(\sigma-1)}\frac{w_i}{P_i^{1/\beta}} & \text{with misallocation} \end{cases}, \tag{2.4}$$

$$\text{where } \tilde{\eta}_i = \frac{\sum_j \mathbf{E}_i\left[\eta r_{ij}(\varphi,\eta)\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right]}{\sum_j \mathbf{E}_i\left[r_{ij}(\varphi,\eta)\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right]}. \tag{2.5}$$

In the case of misallocation, aggregate productivity is adjusted for the inefficient allocation of productive resources across firms. The scaling factor $\tilde{\eta}_i$ represents the size-weighted average distortion η to true firm productivity φ. When there is no misallocation, $\eta = 1$ for all firms and $\tilde{\eta}_i = 1$ drops out.

Since the expression for $\tilde{\Phi}_i$ without misallocation follows directly from that with misallocation, we derive it explicitly for the case of misallocation. The derivation for the case without misallocation is equivalent after replacing φ with $\varphi\eta$.

From the definitions of $\Phi_i(\cdot)$ and $\theta_i(\cdot)$, aggregate productivity can be written as:

$$\tilde{\Phi}_i = \frac{\sum_j \mathbf{E}_i\left[r_{ij}(\varphi,\eta)\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right]}{\sum_j \mathbf{E}_i\left[w_i l_{ij}(\varphi,\eta)\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right]}\frac{w_i}{P_i^{1/\beta}}.$$

Since $r_{ij}(\varphi,\eta) = \left(\frac{\varphi\eta}{\underline{\varphi}_{ij}^*}\right)^{\sigma-1}\sigma w_i f_{ij}$, $w_i l_{ij}(\varphi,\eta) = \frac{\sigma-1}{\sigma}\eta r_{ij}(\varphi,\eta) + f_{ij}$, and $\varphi\eta$ is distributed Pareto with parameters φ_i^m and $\theta > \sigma - 1$, the ex-ante expected average sales and wagebill can be expressed as:

$$\sum_j \mathbf{E}_i\left[r_{ij}(\varphi,\eta)\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right] = \frac{\sigma\theta}{\theta-(\sigma-1)}\sum_j w_i f_{ij}\mathbf{E}_i\left[\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right]$$

and

$$\begin{aligned} \sum_j \mathbf{E}_i\left[w_i l_{ij}(\varphi,\eta)\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right] &= \frac{\sigma-1}{\sigma}\tilde{\eta}_i\sum_j \mathbf{E}_i\left[r_{ij}(\varphi,\eta)\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right] + \sum_j w_i f_{ij}\mathbf{E}_i\left[\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right] \\ &= \frac{(\sigma-1)\theta\tilde{\eta}_i + \theta - (\sigma-1)}{\sigma\theta}\sum_j \mathbf{E}_i\left[r_{ij}(\varphi,\eta)\mathbf{I}(\varphi\eta \geq \underline{\varphi}_{ij}^*)\right]. \end{aligned}$$

[4]In the data, the firm weights are defined such that they sum to 1 across firms. Here, $\theta_i(\varphi)$ and $\theta_i(\varphi,\eta)$ are defined such that they average 1 across firms. This ensures that the residual in the OP decomposition is the covariance of firms' measured productivity and employment share.

Rearranging delivers expression (2.4) for aggregate measured productivity $\tilde{\Phi}_i$.

2.3 Proof of Lemma 2

Lemma 2 in the paper states:

Lemma 2 *Without misallocation, measured aggregate productivity increases with the domestic productivity cut-off,* $\frac{d\tilde{\Phi}_i}{d\varphi^*_{ii}} > 0$. *With misallocation, this relationship becomes ambiguous,* $\frac{d\tilde{\Phi}_i}{d\varphi^*_{ii}} \gtreqless 0$.

This lemma follows directly from Lemma 1 and equations (1.22) and (2.4).

3 Trade Liberalization

In this section, we examine the effects of trade liberalization on welfare and aggregate measured productivity in the three model scenarios introduced above. Both import and export liberalization improve a country's welfare and aggregate productivity in a one-sector frictionless economy. In a two-sector frictionless economy by contrast, bilateral and export liberalizations increase welfare and aggregate productivity, while unilateral import liberalization acts in reverse due to a delocation effect. In the presence of resource misallocation, all three types of trade liberalization have ambiguous effects.

3.1 Efficient allocation and flexible wages: Proof of Proposition 1

Section 2.4.1 in the paper examines the impact of trade liberalization in the case of efficient resource allocation and no outside sector ($\beta = 1$). Its results are summarized by the following proposition:

Proposition 1 *Under no misallocation and flexible wages* ($\beta = 1$)*, bilateral and unilateral trade liberalizations (i.e. reduction in* τ_{ij}, τ_{ji}, *or both* τ_{ij} *and* τ_{ji}*) increase welfare* W_i *and measured aggregate productivity* $\tilde{\Phi}_i$*, but have ambiguous effects on average productivity* $\overline{\Phi}_i$ *and covariance* $\ddot{\Phi}_i$.

Proof. The proof of this proposition builds on an intermediate result summarized in the following lemma:

Appendix Lemma 1 *Under no misallocation and flexible wages* ($\beta = 1$)*, a reduction in the export cost* τ_{12} *or in the import cost* τ_{21} *increases the domestic productivity cut-off* φ^*_{11}.

Equilibrium conditions (2.11), (2.12), (2.13), and (2.14) in the paper can be expressed in terms of the model parameters and endogenous variables $\{\varphi^*_{11}, \varphi^*_{12}, \varphi^*_{21}, \varphi^*_{22}, M_1, M_2, w_1, w_2\}$ with the following system of equations:

$$\left(\frac{\varphi_{21}^*}{\varphi_{11}^*}\right)^{\sigma-1} = \tau_{21}^{\sigma-1}\frac{f_{21}}{f_{11}}\left(\frac{w_2}{w_1}\right)^{\sigma}, \tag{3.1}$$

$$\left(\frac{\varphi_{12}^*}{\varphi_{22}^*}\right)^{\sigma-1} = \tau_{12}^{\sigma-1}\frac{f_{12}}{f_{22}}\left(\frac{w_1}{w_2}\right)^{\sigma}, \tag{3.2}$$

$$L_1 = \frac{\sigma\theta}{\sigma-1}M_1 f_1^E, \tag{3.3}$$

$$L_2 = \frac{\sigma\theta}{\sigma-1}M_2 f_2^E, \tag{3.4}$$

$$f_1^E = \frac{\sigma-1}{\theta-(\sigma-1)}(\varphi_1^m)^{\theta}\left(f_{11}(\varphi_{11}^*)^{-\theta} + f_{12}(\varphi_{12}^*)^{-\theta}\right), \tag{3.5}$$

$$f_2^E = \frac{\sigma-1}{\theta-(\sigma-1)}(\varphi_2^m)^{\theta}\left(f_{21}(\varphi_{21}^*)^{-\theta} + f_{22}(\varphi_{22}^*)^{-\theta}\right), \tag{3.6}$$

$$M_1 w_1 f_{12}(\varphi_1^m)^{\theta}(\varphi_{12}^*)^{-\theta} = M_2 w_2 f_{21}(\varphi_2^m)^{\theta}(\varphi_{21}^*)^{-\theta}. \tag{3.7}$$

Let country 2's labor be the numeraire, such that $w_2 = 1$. The mass of entrants can be determined directly from the labor market clearing conditions (3.3) and (3.4). From the free entry conditions (3.5) and (3.6), φ_{ii} can be expressed as a function of φ_{ij}, denoted $\varphi_{ii} = h_{ii}(\varphi_{ij})$. From the zero cut-off profit conditions (3.1) and (3.2), φ_{ij} can in turn be written as a function of φ_{jj} and w_1, denoted $\varphi_{ij} = k_{ij}(\varphi_{jj}, w_1)$. Thus the system can be reduced to two equations, (3.2) and (3.7), in two unknowns, φ_{12}^* and w_1.

Equation (3.2) implies a positive relationship between φ_{12}^* and w_1:

$$\left(\frac{d\varphi_{12}^*}{dw_1}\right) = \frac{\frac{\partial h_{12}}{\partial w_1} + \frac{\partial h_{12}}{\partial \varphi_{22}^*}\frac{\partial k_{22}}{\partial \varphi_{21}^*}\frac{\partial h_{21}}{\partial w_1}}{1 - \frac{\partial h_{12}}{\partial \varphi_{22}^*}\frac{\partial k_{22}}{\partial \varphi_{21}^*}\frac{\partial h_{21}}{\partial \varphi_{11}^*}\frac{\partial k_{11}}{\partial \varphi_{12}^*}} = \frac{\frac{\sigma}{\sigma-1}\frac{\varphi_{12}^*}{w_1}\left(1 + \frac{f_{21}}{f_{22}}\left(\frac{\varphi_{21}^*}{\varphi_{22}^*}\right)^{-\theta}\right)}{1 - (\tau_{12}\tau_{21})^{1-\sigma}\left(\frac{\varphi_{12}^*\varphi_{21}^*}{\varphi_{11}^*\varphi_{22}^*}\right)^{(\sigma-1)-\theta}} > 0.$$

On the other hand, equation (3.7) implies a negative relationship between φ_{12}^* and w_1.

Rearranging this equation gives:

$$w_1 = \left(\frac{L_2 f_{21} f_1^E \varphi_2^m}{L_1 f_{12} f_2^E \varphi_1^m}\right)\left(\frac{\varphi_{21}^*}{\varphi_{12}^*}\right)^{-\theta}.$$

Substituting for w_1 using (3.1) and rearranging,

$$(\varphi_{21}^*)^{\frac{\sigma-1}{\sigma}-\theta} = \left(\frac{L_1 f_{12} f_2^E \varphi_1^m}{L_2 f_{21} f_1^E \varphi_2^m}\right)\tau_{21}^{\frac{\sigma-1}{\sigma}}\left(\frac{f_{21}}{f_{11}}\right)^{\frac{1}{\sigma}}(\varphi_{11}^*)^{\frac{\sigma-1}{\sigma}}(\varphi_{12}^*)^{-\theta}.$$

The left hand side of this equation is decreasing in φ_{21}^* because $\theta > \sigma - 1$ and $\sigma > 1$ by assumption. The right hand side is decreasing in φ_{12}^*, since the free entry condition (3.5) implies that φ_{11}^* and φ_{12}^* move in opposite directions. Therefore, φ_{12}^* and φ_{21}^* move in the same direction. Condition (3.1) then implies that w_1 and φ_{12}^* move in opposite directions: If w_1 rises, $\varphi_{21}^*/\varphi_{11}^*$ must fall. Since φ_{11}^* and φ_{12}^* move in opposite directions but φ_{12}^* and φ_{21}^* move in the same direction, this can only occur when φ_{21}^* and φ_{12}^* decrease while φ_{11}^* increases.

Therefore, equations (3.2) and (3.7) determine the unique equilibrium (φ_{12}^*, w_1), as illustrated in Appendix Figure 1.

We next examine the impact of trade liberalization by showing that a reduction in the bilateral trade cost τ_{21} decreases φ_{12}^*. From the perspective of country 1, this corresponds to an import liberalization. Recall from the free entry condition (3.5) that the productivity cut-offs for production and for exporting, φ_{11}^* and φ_{12}^*, move in opposite directions. An import reform that reduces the export cut-off φ_{12}^* would thus increase productivity cut-off φ_{11}^*.

From the perspective of country 2, a fall in τ_{21} corresponds to an export liberalization. If φ_{12}^* decreases in response, so would φ_{21}^*, since φ_{12}^* and φ_{21}^* move in the same direction as argued above. Given the free entry condition (3.6), an export reform would then raise productivity cut-off φ_{22}^*.

We illustrate the effect of a reduction in τ_{21} in Appendix Figure 1. This trade cost shock shifts both curves downward. To see this, consider first the curve associated with (3.2). Holding φ_{12}^* fixed, free entry (3.5) implies that φ_{11}^* would also be fixed. From equations (3.1) and (3.2), it follows that $\varphi_{12}^*\varphi_{21}^* = \tau_{12}\tau_{21}\left(\frac{f_{12}f_{21}}{f_{11}f_{22}}\right)^{\frac{1}{\sigma-1}}\varphi_{11}^*\varphi_{22}^*$. So if τ_{21} falls, φ_{22}^* must increase and φ_{21}^* must decrease. From equation (3.2), w_1 would then fall.

Consider next the curve associated with (3.7). Holding w_1^* fixed, we now show that φ_{12}^* would decrease if τ_{21} falls. Since φ_{12}^* and φ_{21}^* move in the same direction, it is sufficient to show that φ_{21}^* must fall. By way of contradiction, suppose φ_{21}^* were to increase. Then (3.1) implies that φ_{11}^* would rise as well. In turn, (3.5) implies that φ_{12}^* would decrease. But then φ_{21}^* would have to fall as well, contradicting the initial assumption.

Since both curves shift down with a reduction in τ_{21}, the wage w_1 must fall. One can further establish that φ_{12}^* must also fall. Suppose by way of contradiction that φ_{12}^* were to rise. Then from (3.2), φ_{22}^* would have to increase, and from (3.6) φ_{21}^* would in turn have to fall. This would, however, violate the result above that φ_{12}^* and φ_{21}^* must move in the same direction.

This completes the proof of Appendix Lemma 1.■

Equation (2.20) in the paper shows that welfare W_i is proportional to the domestic productivity cut-off φ_{ii}^*, where the scaling constant is invariant to trade costs. Equations (2.20) and (2.25) in the paper imply that measured aggregate productivity $\tilde{\Phi}_i$ is proportional to welfare, where the scaling constant is a function of θ and σ alone. The resutls for W_i and $\tilde{\Phi}_i$ in Proposition 1 therefore follow directly from Appendix Lemma 1.

Unlike W_i and $\tilde{\Phi}_i$, average productivity $\overline{\Phi}_i$ and covariance $\ddot{\Phi}_i$ do not have closed-form analytical solutions in terms of trade costs or productivity cut-offs. However, numerical exercises indicate that they can either rise or fall in response to each trade reform considered at different segments of the parameter space. This supports the ambiguous predictions in Proposition 1. ■

3.2 Efficient allocation and fixed wages: Proof of Proposition 2

Section 2.4.1 in the paper examines the impact of trade liberalization in the case of efficient resource allocation and an outside sector ($\beta < 1$). Its results are summarized by the following proposition:

Proposition 2 *Under no misallocation and fixed wages ($\beta < 1$), bilateral and unilateral export liberalizations (i.e. reduction in τ_{ij} or both τ_{ij} and τ_{ji}) increase welfare W_i and measured aggregate productivity $\tilde{\Phi}_i$, but have ambiguous effects on average productivity $\overline{\Phi}_i$ and covariance $\ddot{\Phi}_i$. Unilateral import liberalization (i.e. reduction in τ_{ji}) reduces W_i and $\tilde{\Phi}_i$, but has ambiguous effects on $\overline{\Phi}_i$ and $\ddot{\Phi}_i$.*

Proof. The proof of this proposition builds on an intermediate result summarized in the following lemma:

Appendix Lemma 2 *Under no misallocation and fixed wages ($\beta < 1$), a reduction in the export cost τ_{12} or in bilateral trade costs τ_{12} and τ_{21} increases the domestic productivity cut-off φ^*_{11}, while a reduction in the import cost τ_{21} decreases φ^*_{11}.*

Since wages are fixed, the productivity cut-offs can be determined from the zero cut-off profits and free entry conditions (1.5) and (1.6) alone. Conditions (1.5) for φ^*_{jj} and φ^*_{ij} imply:

$$\frac{\varphi^*_{ij}}{\varphi^*_{jj}} = d_{ij}, \quad d_{ij} \equiv \left(\frac{w_i f_{ij}}{w_j f_{jj}}\right)^{\frac{1}{\sigma-1}} \left(\frac{w_i \tau_{ij}}{w_j \tau_{jj}}\right),$$

while condition (1.6) can be expressed as:

$$\tilde{f}^E_i = \sum_j a_{ij} (\varphi^*_{jj})^{-\theta}, \quad \text{where} \qquad \tilde{f}^E_i \equiv \frac{\theta - (\sigma-1)}{\sigma - 1} f^E_i (\varphi^m_i)^{-\theta} \quad \text{and} \quad a_{ij} \equiv f_{ij} d^{-\theta}_{ij}.$$

Note that a_{ij} measures trade opennness in that it is decreasing in f_{ij} and τ_{ij}.

The equilibrium domestic productivity cut-offs can be determined from:

$$\varphi^{-\theta}_d = A^{-1} \tilde{f}^E,$$

where $\varphi^{-\theta}_d$ is the vector of $(\varphi^*_{ii})^{-\theta}$, A is the square matrix of a_{ij}, and $\tilde{f}^E$ is the vector of $\tilde{f}^E_i$. We assume A is nonsingular to ensure the existence of a unique equilibrium.

Explicitly solving for φ^*_{11} yields:

$$\varphi^*_{11} = \left(\frac{a_{22}\tilde{f}^E_1 - a_{12}\tilde{f}^E_2}{a_{11}a_{22} - a_{12}a_{21}}\right)^{-\frac{1}{\theta}}. \tag{3.8}$$

Clearly, a unilateral import liberalization in country 1 that reduces τ_{21} and thus increases a_{21} will decrease the domestic productivity cut-off φ^*_{11}.

Conversely, a unilateral export liberalization in country 1 that reduces τ_{12} and thus increases a_{12} will likewise raise φ^*_{11}. Taking the derivative of φ^*_{11} with respect to a_{12} gives:

$$\frac{d\varphi^*_{11}}{da_{12}} = \frac{a_{22}}{\theta} \frac{(\varphi^*_{11})^{-\frac{1}{\theta}-1} (\varphi^*_{22})^{-\theta}}{a_{11}a_{22} - a_{12}a_{21}} > 0. \tag{3.9}$$

Finally, a bilateral trade liberalization between two symmetric countries ($\varphi^*_{11} = \varphi^*_{22}, a_{11} = a_{22} = a_d, a_{12} = a_{21} = a_t$) would raise the productivity cut-offs in both countries. To see this, note that a bilateral reduction in $\tau_{12} = \tau_{12} = \tau$ would lower the export cut-offs in both countries, and thereby raise the domestic production cut-offs due to free entry. Formally, the cut-off expression simplifies to:

$$\varphi^*_{11} = \left(\frac{\tilde{f}^E_1}{a_d + a_t} \right)^{-\frac{1}{\theta}}, \tag{3.10}$$

which is clearly increasing in a_t and hence decreasing in τ.■

Equation (2.20) in the paper shows that welfare W_i is proportional to the domestic productivity cut-off φ^*_{ii}, where the scaling constant is invariant to trade costs. Equation (2.25) in the paper shows that aggregate measured productivity $\tilde{\Phi}_i$ is proportional to $P_i^{-1/\beta}$. Since welfare is proportional to $1/P_i$, aggregate productivity must move in the same direction as welfare in response to trade liberalization. The resutls for W_i and $\tilde{\Phi}_i$ in Proposition 2 therefore follow directly from Appendix Lemma 2. As in Proposition 1, the ambiguous predictions for average productivity $\overline{\Phi}_i$ and covariance $\ddot{\Phi}_i$ in Proposition 2 are based on their varying response to trade reforms in numerical simulations with different parameter values. ■

3.3 Resource misallocation: Proof of Proposition 3

Section 2.4.2 in the paper examines the impact of trade liberalization in the case of resource misallocation. Its results are summarized by the following proposition:

Proposition 3 *Under resource misallocation, bilateral and unilateral trade liberalization (i.e. reductions in τ_{ij}, τ_{ji}, or both τ_{ij} and τ_{ji}) have ambiguous effects on welfare W_i, measured aggregate productivity $\tilde{\Phi}_i$, average productivity $\overline{\Phi}_i$, and covariance $\ddot{\Phi}_i$.*

Proof. To prove this proposition, it is sufficient to show that there exists some joint distribution $H_i(\varphi, \eta)$ and model parameters such that a given trade cost shock can either increase or reduce welfare W_i and aggregate productivity $\tilde{\Phi}_i$.

Note from equation (1.21) that welfare W_i depends on trade costs τ_{ij} and τ_{ji} only through their effect on the distorted productivity cut-off for domestic production $\underline{\varphi}^*_{ii}$ and the share of disposable income χ_i; this is implicitly equivalent to the effects on the two cut-offs for domestic production and exporting, $\underline{\varphi}^*_{ii}$ and $\underline{\varphi}^*_{ij}$.

Consider the following $H_i(\varphi, \eta)$ special case. Firms first draw distorted productivity $\underline{\varphi}$ from the Pareto distribution (1.2). Then firms with $\underline{\varphi} \in [\phi - \varepsilon, \phi]$ are assigned $\eta = \bar{\eta} > 1$, while all other firms are assigned $\eta = 1$. True firm productivity is given by $\varphi = \frac{\underline{\varphi}}{\eta}$.

The total lump-sum tax on consumers can be expressed as the sum of the subsidies provided for the domestic and export sales of subsidized firms, $T_i = \sum_j T_{ij}$, where:

$$T_{ij} \equiv \frac{\theta(\sigma - 1)}{\theta - (\sigma - 1)} M_i w_i (\varphi^m_i)^\theta f_{ij} (\bar{\eta} - 1) \left((\phi - \varepsilon)^{-(\theta - (\sigma - 1))} - \phi^{-(\theta - (\sigma - 1))} \right) (\underline{\varphi}^*_{ij})^{-(\sigma - 1)} > 0.$$

Consider two scenarios. Assume first that other model paramaters and initial trade costs are such that $\underline{\varphi}^1_{ii} < \phi - \varepsilon < \phi < \underline{\varphi}^1_{ij}$. Then only some domestic producers but no exporters would be subsidized, and $T^1_i = T^1_{ii}$. Suppose that a trade cost shock pushes down the export cut-off and consequently raises the production cut-off, such that $\underline{\varphi}^2_{ii} < \underline{\varphi}^2_{ij} < \phi - \varepsilon$ after the shock. Now some exporters would receive subsidies, and $T^2_i = T^2_{ii} + T^2_{ij}$. This shows that a marginal reduction in $\underline{\varphi}^*_{ij}$ from ϕ to $\phi - \varepsilon$ can generate a discrete rise in T_i when $\bar{\eta}$ is sufficiently large relative to ε. The concurrent change in $\underline{\varphi}^*_{ii}$ and M_1, however, would be continuous. Therefore, such a trade shock would trigger a discrete drop in χ_i but a marginal rise in $\underline{\varphi}^*_{ii}$, leading to a fall in aggregate welfare W_i.

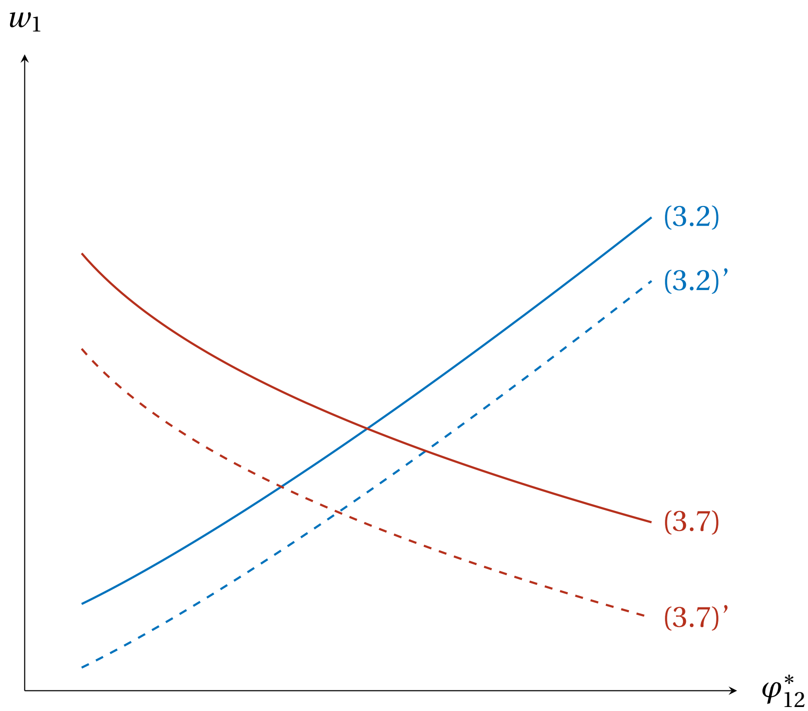
Intuitively, this sample economy subsidizes a small set of firms, that become larger than socially optimal while all other firms remain smaller than optimal due to general equilibrium forces. Trade liberalization can exacerbate this misallocation when it allows firms that are already too large to become even larger by accessing the foreign market, while firms that are already too small become even smaller or exit. This loss due to increased misallocation can outweigh the benefits of trade liberalization and reduce overall welfare.

Assume next that model parameters and initial trade costs are such that $\underline{\varphi}^1_{ii} < \phi - \varepsilon < \phi < \underline{\varphi}^1_{ij}$. Suppose that a trade cost shock pushes down the export cut-off and consequently raises the production cut-off, such that $\underline{\varphi}^2_{ii} < \phi - \varepsilon < \phi < \underline{\varphi}^2_{ij}$ continues to hold after the shock. Now a subset of domestic producers and no exporters would receive subsidies both before and after the shock, and the total value of these subsidies would moreover fall as producers contract domestic sales, $T^2_i = T^2_{ii} < T^1_i = T^1_{ii}$. Now a marginal reduction in $\underline{\varphi}^*_{ij}$ would generate a marginal fall in T_i and a marginal rise in $\underline{\varphi}^*_{ii}$. Such a trade shock would thus increase aggregate welfare W_i.

A similar argument applies to aggregate productivity $\tilde{\Phi}_i$. The effects of trade cost shocks $\tilde{\Phi}_i$ can be assessed based on equation (2.4). In the first scenario above for example, the sales-weighted average subsidy rate $\tilde{\eta}_i$ would increase discretely when the export cut-off $\underline{\varphi}^*_{ij}$ falls below $\phi - \varepsilon$. The consumer price index P_i, however, would decrease continuously in $\underline{\varphi}^*_{ii}$. Therefore, $\tilde{\Phi}_i$ would fall if $\bar{\eta}$ is sufficiently large. Conversely, $\tilde{\Phi}$ would rise in the latter scenario.

As in the absence of misallocation, average productivity $\overline{\Phi}_i$ and covariance $\ddot{\Phi}_i$ under misallocation do not receive closed-form analytical solutions in terms of trade costs or productivity cut-offs. Unlike the case of efficient allocation, the effects of trade reforms on W_i and $\tilde{\Phi}_i$ are ambiguous with distortions. It is thus less surprising that numerical exercises reveal ambiguous effects of trade reforms on $\overline{\Phi}_i$ and $\ddot{\Phi}_i$as well. ■

Figure 1: Equilibrium export cutoff and wage ($\beta = 1$)



Note: The diagram illustrates the relationship between country 1's wage w_1 and export cutoff φ^*_{12} as given by zero cutoff profit condition (3.2) and the balanced trade condition (3.7). The equilibrium level of (w_1, φ^*_{12}) is determined at the intersection of the two curves. The dashed lines show the shift in the relationships due to a reduction in import cost τ_{21}, which reduces the equilibrium wage w_1 and the export cutoff φ^*_{12}.