Host-Country Financial Development and Multinational Activity: Model Appendix

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Abstract

In this Appendix, we spell out details of our model set-up and provide the proofs for the lemmas and propositions from Sections 2.3 and 2.4 of the main paper. In particular, we show how the predictions of the model inform our empirical analysis, in terms of: (i) the mapping between the theoretical and observed measures of financial development; and (ii) the effects of host-country financial development and cross-industry variation in external finance dependence on the MNC outcomes of interest. In the second part of the Appendix, we document how the *competition effect* and the *financing effect* of host-country financial development on MNC activity operate in several extensions to more general setups. These incorporate respectively: (i) home-bias in consumption; (ii) exports by host-country firms in South; (iii) endogenous host-country wages; and (iv) multiple FDI host countries.

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A Model Details and Proofs

Setup. The utility function of the representative consumer in West and East (subscript $n = w, e$) is given by:

$$U_n = (y_n)^{\mu^0} \prod_{k=1}^{K} \left(\sum_{j \in \{e,w\}} \int_{\Omega_{nj}^k} x_{nj}^k(a)^{\alpha^k} \, dG_j^k(a) \right)^{\frac{\mu^k}{\alpha^k}}, \tag{A.1}$$

while the utility function for Southern consumers (subscript s) is:

$$U_s = (y_s)^{\mu^0} \prod_{k=1}^{K} \left(\sum_{j \in \{e,w,s\}} \int_{\Omega_{sj}^k} x_{sj}^k(a)^{\alpha^k} \, dG_j^k(a) \right)^{\frac{\mu^k}{\alpha^k}}, \tag{A.2}$$

where $0 < \alpha^k, \mu^k < 1$ and $\sum_{k=0}^{K} \mu^k = 1$. Utility in country i ($i \in \{w, e, s\}$) is thus a Cobb-Douglas function over consumption of the homogeneous good (y_i) and differentiated varieties ($x_{ij}^k(a)$, indexed by a), where the expenditure share on industry k is equal to μ^k. The sub-utility derived from industry $k = 1, \ldots, K$ features a constant elasticity of substitution $\varepsilon^k = \frac{1}{1-\alpha^k} > 1$ across varieties from all countries of origin. We denote by $x_{ij}^k(a)$ the quantity of a industry-k variety consumed in country i that originates from a country-j firm, and label the set of such varieties Ω_{ij}^k. When $i \neq j$, Ω_{ij}^k consists of all country-j varieties exported to i, as well as varieties sold locally by country j's multinational affiliates located in i. On the other hand, when $i = j$, Ω_{ii}^k consists of all domestic varieties produced at home, and all varieties produced by country i's multinational affiliates abroad that are then exported back to the home market. Note from (A.1) that Southern varieties do not enter the utility function in West and East; this simplifies our analysis but does not detract from the main effects below. (We consider the case where South can export its varieties later in this Online Appendix.)

As is well-known, the above preferences imply that demand in country i for each variety is: $x_{ij}^k(a) = A_{ij}^k (p_{ij}^k(a))^{-\varepsilon^k}$, where $p_{ij}^k(a)$ is the price of that variety. In turn, the aggregate demand levels, A_{ij}^k, are given by the following expressions:

$$A_{ww}^k = A_{ee}^k = A_{ew}^k = A_{we}^k = \frac{\mu^k E_n}{(P_{ww}^k)^{1-\varepsilon^k} + (P_{we}^k)^{1-\varepsilon^k}}, \quad \text{and} \tag{A.3}$$

$$A_{sw}^k = A_{se}^k = A_{ss}^k = \frac{\mu^k E_s}{(P_{ss}^k)^{1-\varepsilon^k} + 2(P_{sw}^k)^{1-\varepsilon^k}}, \tag{A.4}$$

where $(P_{ij}^k)^{1-\varepsilon^k} = \int_{\Omega_{ij}^k} p_{ij}^k(a)^{1-\varepsilon^k} dG_j^k(a)$ is the ideal price index for country-j varieties from industry k that are consumed in country i. Here, E_i is total expenditure in i, which is pinned down by aggregate labor income in that country. Since West and East are symmetric, we have $E_w = E_e = E_n$, as well as $P_{ww}^k = P_{ee}^k$, $P_{ew}^k = P_{we}^k$, and $P_{sw}^k = P_{se}^k$; it follows that $A_{ww}^k = A_{ee}^k = A_{ew}^k = A_{we}^k$ and $A_{sw}^k = A_{se}^k = A_{ss}^k$. ■

The FDI decision. We consider the decision faced by a firm in a given differentiated-varieties industry; we thus drop the sector superscript k in what follows to ease the notation. We show that the two conditions, $\tau\omega < 1$ and $f_X < f_D < f_I$, are sufficient to guarantee that the optimal strategy for Western multinationals will be as follows: (i) highly productive Western firms conduct FDI only in South but not in East; and (ii) Western multinationals use their Southern plant as a global production center to serve all three markets.

Consider first a Western firm that already operates a multinational affiliate in South. It is then automatically more profitable to use this affiliate as an export platform to East, rather than servicing East via direct exports from West, or via direct FDI in East. This follows from the inequality:

$$(1-\alpha)A_{ew}\left(\frac{\tau a\omega}{\alpha}\right)^{1-\varepsilon} - f_X \;>\; \max\left\{(1-\alpha)A_{ew}\left(\frac{\tau a}{\alpha}\right)^{1-\varepsilon} - f_X,\; (1-\alpha)A_{ew}\left(\frac{a}{\alpha}\right)^{1-\varepsilon} - f_I\right\},$$

which holds since $\tau\omega < 1 < \tau$ and $f_X < f_I$ (bearing in mind that $1-\varepsilon < 0$). In particular, this rules out the possibility of the MNC establishing affiliates in both South and East.

Next, conditional on setting up a Southern affiliate, we can further deduce that it is optimal to use this affiliate to supply even the firm's home market. This follows from:

$$(1-\alpha)A_{ww}\left(\frac{\tau a\omega}{\alpha}\right)^{1-\varepsilon} - f_X \;>\; (1-\alpha)A_{ww}\left(\frac{a}{\alpha}\right)^{1-\varepsilon} - f_D,$$

which holds since $\tau\omega < 1$ and $f_X < f_D$. Thus, it is more profitable to produce in South and export to West than to incur the higher fixed cost and wages of production at home.

It remains to check that the optimal decision for a Western firm that becomes a multinational is to locate its overseas affiliate in South, rather than in East. For this, we compare the total profits from servicing all three countries out of an affiliate in South versus the total profits from setting up an affiliate in East:

$$\begin{aligned}
&(1-\alpha)A_{ww}\left(\frac{\tau a\omega}{\alpha}\right)^{1-\varepsilon} - f_X + (1-\alpha)A_{ew}\left(\frac{\tau a\omega}{\alpha}\right)^{1-\varepsilon} - f_X + (1-\alpha)A_{sw}\left(\frac{a\omega}{\alpha}\right)^{1-\varepsilon} - f_I \\
&> \max\left\{(1-\alpha)A_{ww}\left(\frac{a}{\alpha}\right)^{1-\varepsilon} - f_D,\; (1-\alpha)A_{ww}\left(\frac{\tau a}{\alpha}\right)^{1-\varepsilon} - f_X\right\} \\
&\quad + (1-\alpha)A_{ew}\left(\frac{a}{\alpha}\right)^{1-\varepsilon} - f_I + (1-\alpha)A_{sw}\left(\frac{\tau a}{\alpha}\right)^{1-\varepsilon} - f_X.
\end{aligned}$$

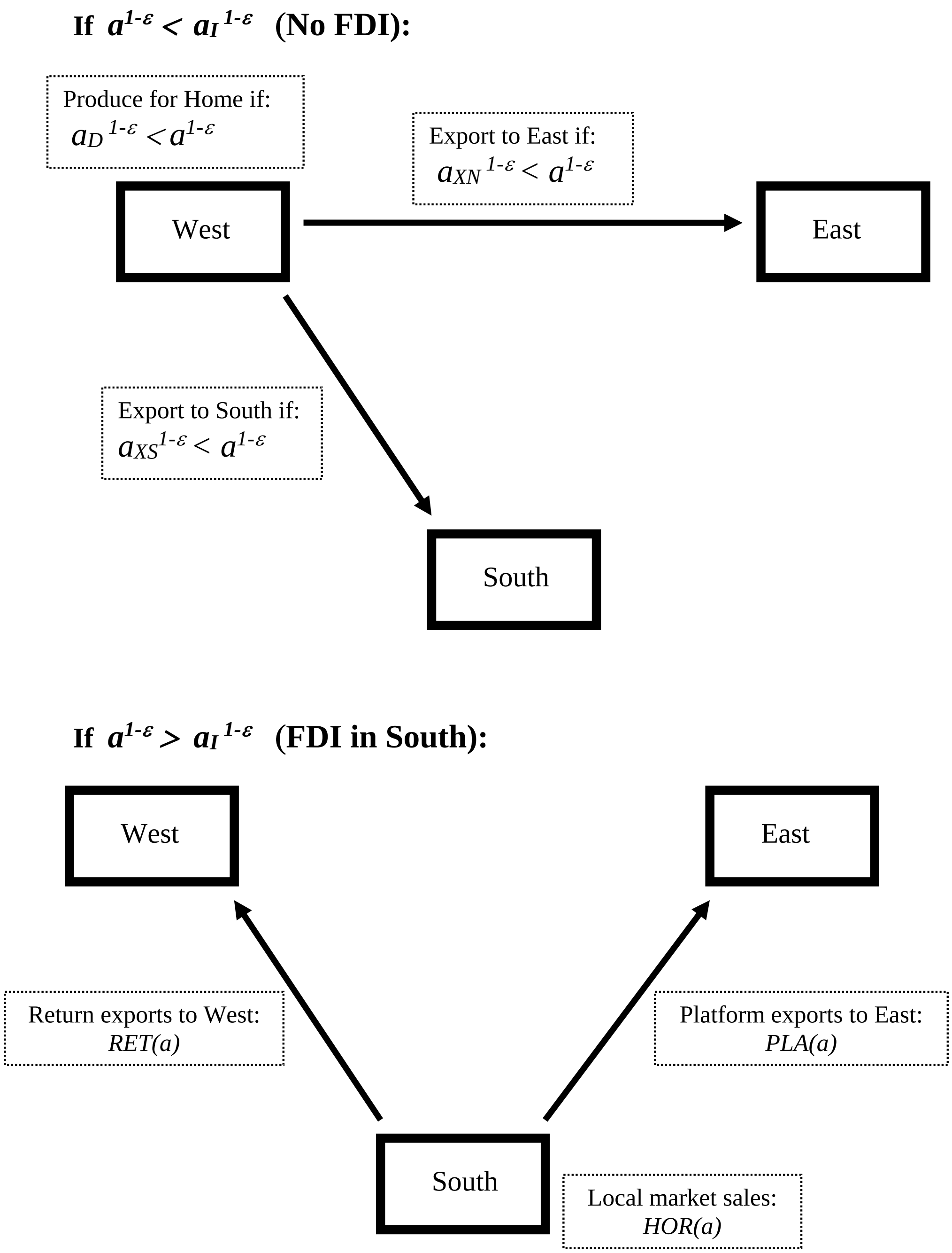
Note that if FDI is undertaken in East, we do consider the possibility that the home market (West) can be supplied either through domestic production or exports from East, while South would be serviced by exports from either West or East. The expression on the right-hand side of the above inequality captures total profits from this alternative production mode. It is straightforward to check that the above inequality holds when $\tau\omega < 1$, $\omega < 1$, $\omega < \tau$ and $f_X < f_D$. It is thus not optimal for a Western firm to conduct FDI in East.

In sum, the conditions $\tau\omega < 1$ and $f_X < f_D < f_I$ guarantee that the FDI decision is in effect a decision over whether to relocate the firm's global production center to South, with only headquarter activities retained in West. ■

Industry Equilibrium. Once again, we focus the exposition on the equilibrium in a given heterogeneous-firm industry, so we drop the sector superscript k. In equilibrium, we assume that the underlying parameters of the model are such that the following ordering of productivity cut-offs is satisfied: $0 < a_D^{1-\varepsilon} < a_{XN}^{1-\varepsilon} < a_{XS}^{1-\varepsilon} < a_I^{1-\varepsilon}$. The condition $a_D^{1-\varepsilon} < a_{XN}^{1-\varepsilon}$ reduces to $\tau^{\varepsilon-1}f_X > f_D$, so that export costs must be sufficiently bigger than the fixed cost of domestic production. The condition $a_{XN}^{1-\varepsilon} < a_{XS}^{1-\varepsilon}$ boils down to a requirement that market demand for Western varieties be greater in East than in South, $A_{ew} > A_{sw}$. The condition $a_{XS}^{1-\varepsilon} < a_I^{1-\varepsilon}$ is satisfied when the fixed cost of FDI is sufficiently high. As explained in the main paper, this ordering of the productivity cut-offs captures the feature that exporters are on average more productive than non-exporters, and that multinational firms tend in turn to be more productive than exporting firms.

Figure A-1 below illustrates the sorting pattern of firms in the industry equilibrium we consider: Firms with $a^{1-\varepsilon} < a_I^{1-\varepsilon}$ base their production activity in West, and export to East and possibly also to South if they are productive enough. The most efficient firms with $a^{1-\varepsilon} > a_I^{1-\varepsilon}$ instead become multinationals; while still headquartered in West, these firms locate production in South and serve all three markets from there.

Figure A-1
Modes of Operation (illustrated for Western firms)



As discussed towards the end of Section 2.2 in the main paper, the industry equilibrium in each sector is closed by writing down the free-entry conditions in West and South for the sector in question. We now spell out the details of these free-entry conditions.

In country j, prospective entrants into the differentiated-varieties sector incur an upfront entry cost equal to f_{Ej} units of country j labor. This is a once-off cost that firms pay before they can obtain their productivity draw.[1] On the exit side, firms face an exogenous probability, $\delta \in (0,1)$, of "dying" and leaving the industry in each period. For an equilibrium with a constant measure of firms in each country, the cost of entry must equal expected profits. Using the profit functions and the productivity cut-offs from Section 2.2 of the main paper, and integrating the expressions for expected profits over the distribution $G_j(a)$, one can write down the free-entry conditions for Western/Eastern ($n = w, e$) and Southern firms as:

$$\begin{aligned}
\delta f_{En} \quad &= \quad (1-\alpha)A_{ww}\left(\frac{1}{\alpha}\right)^{1-\varepsilon}(V_n(a_D) - V_n(a_I)) - f_D(G_n(a_D) - G_n(a_I)) \\
&\quad +(1-\alpha)A_{ew}\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XN}) - V_n(a_I)) - f_X(G_n(a_{XN}) - G_n(a_I)) \\
&\quad +(1-\alpha)A_{sw}\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XS}) - V_n(a_I)) - f_X(G_n(a_{XS}) - G_n(a_I)) \\
&\quad +(1-\alpha)\left(A_{ww}\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} + A_{ew}\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} + A_{sw}\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}\right)V_n(a_I) - (f_I + 2f_X)G_n(a_I), \quad \text{and}
\end{aligned} \tag{A.5}$$

$$\delta f_{Es}\omega \quad = \quad (1-\alpha)A_{ss}\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}V_s(a_S) - (R\phi + (1-\phi))f_S\omega G_s(a_S). \tag{A.6}$$

Next, we denote the measure of firms in country j's differentiated-varieties sector by N_j.[2] The definition of the ideal price index then implies:

$$P_{ww}^{1-\varepsilon} \quad = \quad N_n\left[\left(\frac{1}{\alpha}\right)^{1-\varepsilon}(V_n(a_D) - V_n(a_I)) + \left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon}V_n(a_I)\right], \tag{A.7}$$

$$P_{ew}^{1-\varepsilon} \quad = \quad N_n\left[\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XN}) - V_n(a_I)) + \left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon}V_n(a_I)\right], \tag{A.8}$$

$$P_{sw}^{1-\varepsilon} \quad = \quad N_n\left[\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XS}) - V_n(a_I)) + \left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}V_n(a_I)\right], \quad \text{and} \tag{A.9}$$

$$P_{ss}^{1-\varepsilon} \quad = \quad N_s\left[\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}V_s(a_S)\right]. \tag{A.10}$$

The equilibrium of the model is thus pinned down by a system of equations comprising: (A.3)-(A.4); the four productivity cut-offs for Western firms respectively for domestic production, exporting to South, exporting to East, and for FDI (presented in Section 2.2 of the main paper); the productivity cut-off for Southern firms (2.5); the free entry conditions (A.5)-(A.6); and the price indices (A.7)-(A.10). There are 13 unknowns in this system, $A_{ww}(= A_{ew})$, $A_{sw}(= A_{ss})$, a_D, a_{XN}, a_{XS}, a_I, a_S, N_n, N_s, P_{ww}, P_{ew}, P_{sw} and P_{ss}. While we cannot solve for all of these variables in closed form, we are able to derive comparative statics results that directly inform our empirical analysis. We assume that productivity $1/a$ follows a Pareto distribution with shape parameter κ and support $[1/\bar{a}_j, \infty)$ for varieties originating from country j.[3] The expressions for G_j and V_j associated with this Pareto distribution are thus: $G_j(a) = \left(\frac{a}{\bar{a}_j}\right)^{\kappa}$ and $V_j(a) = \frac{\kappa}{\kappa-\varepsilon+1}\left(\frac{a^{\kappa-\varepsilon+1}}{\bar{a}_j^{\kappa}}\right)$. We adopt the standard assumption that $\kappa > \varepsilon - 1$, which ensures that the distribution of firm sales has a finite variance. ∎

Proof of Lemma 1. The proofs below hold separately for each differentiated-varieties sector k, and so we omit the sector superscript unless it is necessary for clarity.

[1] Our results are robust to subjecting the fixed cost of entry in South, f_{Es}, to borrowing constraints too. Intuitively, an improvement in financial development in South would still spur more entry by Southern firms, which works in the same direction as the effects in our baseline model.

[2] Following Melitz (2003), for N_j to be constant, the expected mass of successful entrants, N_j^{ent}, needs to equal the mass of firms that dies exogenously in each period, namely: $N_j^{ent} = \delta N_j$, for $j = w, e, s$.

[3] We require that $\bar{a}_s$ and $\bar{a}_n$ both be sufficiently large, so that all relevant cut-offs lie within the interior of the support of the distributions that they are drawn from. Also, our proofs do not require the same shape parameter in West and South, but we have assumed this to simplify notation.

Log-differentiating equations (2.5) and (A.6), one obtains:

$$
\begin{aligned}
(\varepsilon-1)\frac{da_S}{a_S} &= \frac{dA_{ss}}{A_{ss}} + \frac{d\eta}{\eta} - \frac{d\phi}{\phi}, \quad \text{and} \\
0 &= a_S^{\varepsilon-1}V_s(a_S)\frac{dA_{ss}}{A_{ss}} + \left[a_S^{\varepsilon-1}V_s'(a_S) - \frac{R\phi+(1-\phi)}{R\phi}\eta G_s'(a_S)\right]da_S - \frac{R-1}{R}\frac{\eta}{\phi}G_s(a_S)d\phi.
\end{aligned}
$$

To derive the second equation above, we used the fact that: $(1-\alpha)A_{ss}(\omega/\alpha)^{1-\varepsilon} = (R\phi/\eta)a_S^{\varepsilon-1}f_S\omega$, which holds from the expression for $a_S^{1-\varepsilon}$ in (2.5). Applying Leibniz's rule to $V_s(a_S) = \int_0^{a_S} a^{1-\varepsilon}dG_s(a)$, we have: $a_S^{\varepsilon-1}V_s'(a_S) = G_s'(a_S)$. We substitute this into the system of equations above to get:

$$
\begin{aligned}
(\varepsilon-1)\frac{da_S}{a_S} &= \frac{dA_{ss}}{A_{ss}} + \frac{d\eta}{\eta} - \frac{d\phi}{\phi}, \quad \text{and} \\
0 &= a_S^{\varepsilon-1}V_s(a_S)\frac{dA_{ss}}{A_{ss}} + \left[1 - \frac{R\phi+(1-\phi)}{R\phi}\eta\right]a_S G_s'(a_S)\frac{da_S}{a_S} - \frac{R-1}{R}\eta G_s(a_S)\frac{d\phi}{\phi}.
\end{aligned}
$$

Solving these two equations simultaneously yields:

$$
\begin{aligned}
\frac{da_S}{a_S} &= \frac{a_S^{\varepsilon-1}V_s(a_S)\frac{d\eta}{\eta} + \left[\frac{R-1}{R}\eta G_s(a_S) - a_S^{\varepsilon-1}V_s(a_S)\right]\frac{d\phi}{\phi}}{a_S^{\varepsilon-1}V_s(a_S)(\varepsilon-1) + \left[1-\frac{R\phi+(1-\phi)}{R\phi}\eta\right]a_S G_s'(a_S)}, \text{ and} \\
\frac{dA_{ss}}{A_{ss}} &= \frac{-\left[1-\frac{R\phi+(1-\phi)}{R\phi}\eta\right]a_S G_s'(a_S)\frac{d\eta}{\eta} + \left[\frac{R-1}{R}\eta G_s(a_S)(\varepsilon-1) + \left(1-\frac{R\phi+(1-\phi)}{R\phi}\eta\right)a_S G_s'(a_S)\right]\frac{d\phi}{\phi}}{a_S^{\varepsilon-1}V_s(a_S)(\varepsilon-1) + \left[1-\frac{R\phi+(1-\phi)}{R\phi}\eta\right]a_S G_s'(a_S)}.
\end{aligned}
$$

Recall that we assume: $R\frac{\phi}{1-\phi} > \frac{\eta}{1-\eta}$, which is equivalent to: $1 - \frac{R\phi+(1-\phi)}{R\phi}\eta > 0$. Bearing in mind that $V_s(a_S) > 0$ and $G_s'(a_S) > 0$ for any cdf $G_s(\cdot)$, the denominators in the above expressions for $\frac{da_S}{a_S}$ and $\frac{dA_{ss}}{A_{ss}}$ are thus both positive. Furthermore, in the expression for $\frac{dA_{ss}}{A_{ss}}$, we clearly have the term in $\frac{d\eta}{\eta}$ being negative and the term in $\frac{d\phi}{\phi}$ being positive, so that $\frac{dA_{ss}}{d\eta} < 0$ and $\frac{dA_{ss}}{d\phi} > 0$. Turning to the expression for $\frac{da_S}{a_S}$, by an analogous logic, we also have: $\frac{da_S}{d\eta} > 0$. As for $\frac{da_S}{d\phi}$, observe that:

$$
\begin{aligned}
\frac{R-1}{R}\eta G_s(a_S) - a_S^{\varepsilon-1}V_s(a_S) &= \frac{R-1}{R}\eta G_s(a_S) - a_S^{\varepsilon-1}\int_0^{a_S} a^{1-\varepsilon}dG_s(a) \\
&= \frac{R-1}{R}\eta G_s(a_S) - a_S^{\varepsilon-1}\left[a_S^{1-\varepsilon}G_s(a_S) - \int_0^{a_S} G_s(a)(1-\varepsilon)a^{-\varepsilon}da\right] \\
&= \left(\frac{R-1}{R}\eta - 1\right)G_s(a_S) - (\varepsilon-1)a_S^{\varepsilon-1}\int_0^{a_S} G_s(a)a^{-\varepsilon}da \\
&< 0,
\end{aligned}
$$

where the last step follows from the fact that with $R\frac{\phi}{1-\phi} > \frac{\eta}{1-\eta}$, we have: $\frac{R-1}{R}\eta < \frac{(R-1)\phi}{R\phi+(1-\phi)} < 1$. It thus follows that: $\frac{da_S}{d\phi} < 0$. Note in particular that we have now established the two comparative statics claimed in the lemma, namely: $\frac{da_S}{d\eta} > 0$ and $\frac{dA_{ss}}{d\eta} < 0$. (The results derived for $\frac{da_S}{d\phi}$ and $\frac{dA_{ss}}{d\phi}$ will be used in later proofs.)

While the above proof holds for any cdf $G_s(a)$, it is straightforward to show for the case of the Pareto distribution, $G_s(a) = (a/\bar{a}_s)^\kappa$, that the above derivatives can be further simplified to:

$$
\frac{1}{a_S}\frac{da_S}{d\eta} = \frac{1}{\eta}\frac{1-\rho_S}{\varepsilon-1} > 0, \tag{A.11}
$$

$$
\frac{1}{A_{ss}}\frac{dA_{ss}}{d\eta} = -\frac{1}{\eta}\rho_S < 0, \tag{A.12}
$$

$$
\frac{1}{a_S}\frac{da_S}{d\phi} = \frac{1}{\phi}\frac{1-\rho_S}{\varepsilon-1}\left[-1 + \frac{R-1}{R}\eta\frac{k-\varepsilon+1}{k}\right] < 0, \text{ and} \tag{A.13}
$$

$$
\frac{1}{A_{ss}}\frac{dA_{ss}}{d\phi} = \frac{1}{\phi}\left[\rho_S + \frac{R-1}{R}\eta\frac{k-\varepsilon+1}{k}(1-\rho_S)\right] > 0. \tag{A.14}
$$

Here, ρ_S is a constant that depends only on parameter values: $\rho_S \equiv \frac{\left(1-\frac{R\phi+(1-\phi)}{R\phi}\eta\right)\frac{\kappa-\varepsilon+1}{\varepsilon-1}}{1+\left(1-\frac{R\phi+(1-\phi)}{R\phi}\eta\right)\frac{\kappa-\varepsilon+1}{\varepsilon-1}} \in (0,1)$. These are expressions that we will use frequently to facilitate the rest of the proofs. ■

Proof of Lemma 2. We take the equations that define the industry equilibrium in West and differentiate them. First, recall that the productivity cutoffs in West are given by:

$$a_D^{1-\varepsilon} = \frac{f_D}{(1-\alpha)A_{ww}(1/\alpha)^{1-\varepsilon}} \tag{A.15}$$

$$a_{XN}^{1-\varepsilon} = \frac{f_X}{(1-\alpha)A_{ew}(\tau/\alpha)^{1-\varepsilon}}, \text{ and} \tag{A.16}$$

$$a_{XS}^{1-\varepsilon} = \frac{f_X}{(1-\alpha)A_{sw}(\tau/\alpha)^{1-\varepsilon}}, \tag{A.17}$$

and:

$$a_I^{1-\varepsilon} = \frac{f_I - f_D}{(1-\alpha)[A_{ww}((\frac{\tau\omega}{\alpha})^{1-\varepsilon} - (\frac{1}{\alpha})^{1-\varepsilon}) + A_{ew}((\frac{\tau\omega}{\alpha})^{1-\varepsilon} - (\frac{\tau}{\alpha})^{1-\varepsilon}) + A_{sw}((\frac{\omega}{\alpha})^{1-\varepsilon} - (\frac{\tau}{\alpha})^{1-\varepsilon})]}. \tag{A.18}$$

Log-differentiating (A.15)-(A.17) yields:

$$(\varepsilon-1)\frac{da_D}{a_D} = \frac{dA_{ww}}{A_{ww}}, \tag{A.19}$$

$$(\varepsilon-1)\frac{da_{XN}}{a_{XN}} = \frac{dA_{ew}}{A_{ew}}, \quad \text{and} \tag{A.20}$$

$$(\varepsilon-1)\frac{da_{XS}}{a_{XS}} = \frac{dA_{sw}}{A_{sw}}. \tag{A.21}$$

Since $A_{sw} = A_{ss}$, it immediately follows from (A.12) and (A.21) that $\frac{dA_{sw}}{d\eta} = \frac{dA_{ss}}{d\eta} < 0$, and hence that:

$$\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} = -\frac{1}{\eta}\frac{\rho_S}{\varepsilon-1} < 0. \tag{A.22}$$

This establishes part (iii) of the lemma.

We next differentiate the free-entry condition for West, (A.5):

$$\begin{aligned}
0 = &\left[(1-\alpha)A_{ww}\left(\left(\frac{1}{\alpha}\right)^{1-\varepsilon}V_n(a_D) + \left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right)V_n(a_I)\right)\right]\frac{dA_{ww}}{A_{ww}} \\
&+\left[(1-\alpha)A_{ew}\left(\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}V_n(a_{XN}) + \left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)V_n(a_I)\right)\right]\frac{dA_{ew}}{A_{ew}} \\
&+\left[(1-\alpha)A_{sw}\left(\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}V_n(a_{XS}) + \left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)V_n(a_I)\right)\right]\frac{dA_{sw}}{A_{sw}} \\
&+\left[(1-\alpha)A_{ww}\left(\frac{1}{\alpha}\right)^{1-\varepsilon}V_n'(a_D) - f_D G_n'(a_D)\right]da_D \\
&+\left[(1-\alpha)A_{ew}\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}V_n'(a_{XN}) - f_X G_n'(a_{XN})\right]da_{XN} \\
&+\left[(1-\alpha)A_{sw}\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}V_n'(a_{XS}) - f_X G_n'(a_{XS})\right]da_{XS} \\
&+\Bigg[(1-\alpha)\Bigg(A_{ww}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right) + A_{ew}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right) \\
&+A_{sw}\left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)\Bigg)V_n'(a_I) - (f_I - f_D)G_n'(a_I)\Bigg]da_I.
\end{aligned} \tag{A.23}$$

Focus first on the term involving da_D on the right-hand side of (A.23). We make use of the fact that: (i) $(1-\alpha)A_{ww}(1/\alpha)^{1-\varepsilon} = a_D^{\varepsilon-1}f_D$, which comes from equation (A.15); and (ii) $a^{\varepsilon-1}V_n'(a) = G_n'(a)$ for all $a \in$

$(0, \bar{a}_n)$, which holds from Leibniz's Rule. With these, one can show that the coefficient of da_D in (A.23) reduces to 0. An analogous argument implies that the coefficients of da_{XN}, da_{XS} and da_I are all also equal to 0. Turning to the terms involving $\frac{dA_{ww}}{A_{ww}}$, $\frac{dA_{ew}}{A_{ew}}$ and $\frac{dA_{sw}}{A_{sw}}$, one can use the expressions for the price indices in (A.7)-(A.9) to re-write (A.23) as:

$$\rho_1 \frac{dA_{ww}}{A_{ww}} + (1-\rho_1)\frac{dA_{ew}}{A_{ew}} + \frac{1-\rho_2}{2}\frac{E_s}{E_n}\frac{dA_{sw}}{A_{sw}} = 0,$$

where we define: $\rho_1 = \frac{P_{ww}^{1-\varepsilon}}{P_{ww}^{1-\varepsilon}+P_{ew}^{1-\varepsilon}}$ and $\rho_2 = \frac{P_{ss}^{1-\varepsilon}}{P_{ss}^{1-\varepsilon}+2P_{sw}^{1-\varepsilon}}$. Note that $\rho_1, \rho_2 \in (0,1)$. A quick substitution from (A.19)-(A.21) then implies:

$$\rho_1 \frac{da_D}{a_D} + (1-\rho_1)\frac{da_{XN}}{a_{XN}} + \frac{1-\rho_2}{2}\frac{E_s}{E_n}\frac{da_{XS}}{a_{XS}} = 0. \tag{A.24}$$

Intuitively, the free-entry condition requires that a rise in demand in any one market for the Western firm's goods must be balanced by a decline in demand from at least one other market. Since $A_{ww} = A_{ew}$, we have $\frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} = \frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta}$, and hence from (A.19) and (A.20), we have $\frac{1}{a_D}\frac{da_D}{d\eta} = \frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}$. Substituting this and the expression for $\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta}$ from (A.22) into (A.24), we obtain:

$$\frac{1}{a_D}\frac{da_D}{d\eta} = \frac{1}{a_{XN}}\frac{da_{XN}}{d\eta} = \frac{1}{\eta}\frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\rho_S}{\varepsilon - 1} > 0. \tag{A.25}$$

It follows from (A.19) and (A.20) that: $\frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} = \frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} > 0$, establishing parts (ii) and (iv) of the lemma.

Finally, we turn to part (i) in the statement of Lemma 2. Log-differentiating (A.18) yields:

$$(\varepsilon-1)\frac{da_I}{a_I} = \frac{A_{ww}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right)\frac{dA_{ww}}{A_{ww}} + A_{ew}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)\frac{dA_{ew}}{A_{ew}} + A_{sw}\left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)\frac{dA_{sw}}{A_{sw}}}{A_{ww}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right) + A_{ew}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right) + A_{sw}\left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)}.$$

We replace $\frac{dA_{ww}}{A_{ww}}$, $\frac{dA_{ew}}{A_{ew}}$ and $\frac{dA_{sw}}{A_{sw}}$ using (A.19)-(A.21). Making use also of the expressions: (i) for A_{ww}, A_{ew} and A_{sw} from (A.3)-(A.4); and (ii) for $P_{ww}^{1-\varepsilon}$, $P_{ew}^{1-\varepsilon}$ and $P_{sw}^{1-\varepsilon}$ from (A.7)-(A.9); and simplifying extensively, one can show that:

$$\frac{da_I}{a_I} = \frac{\rho_1(1-\Delta_1)\frac{da_D}{a_D} + (1-\rho_1)(1-\Delta_2)\frac{da_{XN}}{a_{XN}} + \frac{1-\rho_2}{2}\frac{E_s}{E_n}(1-\Delta_3)\frac{da_{XS}}{a_{XS}}}{\rho_1(1-\Delta_1) + (1-\rho_1)(1-\Delta_2) + \frac{1-\rho_2}{2}\frac{E_s}{E_n}(1-\Delta_3)}, \tag{A.26}$$

where we define:

$$\begin{aligned}
\Delta_1 &= \frac{\left(\frac{1}{\alpha}\right)^{1-\varepsilon} V_n(a_D)}{\left(\frac{1}{\alpha}\right)^{1-\varepsilon} V_n(a_D) + (\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon})V_n(a_I)}, \\
\Delta_2 &= \frac{\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} V_n(a_{XN})}{\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} V_n(a_{XN}) + (\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon})V_n(a_I)}, \quad \text{and} \\
\Delta_3 &= \frac{\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} V_n(a_{XS})}{\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} V_n(a_{XS}) + (\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon})V_n(a_I)}.
\end{aligned}$$

Thus, $\frac{da_I}{a_I}$ is a weighted average of $\frac{da_D}{a_D}$, $\frac{da_{XN}}{a_{XN}}$ and $\frac{da_{XS}}{a_{XS}}$. Note that $\Delta_1, \Delta_2, \Delta_3 \in (0,1)$. Moreover, using the above definitions, we have: $sign\{\Delta_1 - \Delta_2\} = sign\{(\omega^{1-\varepsilon} - 1)V_N(a_D) - ((\tau\omega)^{1-\varepsilon} - 1)V_N(a_{XN})\} > 0$. This inequality holds as: $V_N(a_D) > V_N(a_{XN}) > 0$ (since $a_D > a_{XN}$), and $\omega^{1-\varepsilon} - 1 > (\tau\omega)^{1-\varepsilon} - 1 > 0$. Analogously, we have: $sign\{\Delta_2 - \Delta_3\} = sign\{(\omega^{1-\varepsilon} - \tau^{1-\varepsilon})V_N(a_{XN}) - ((\tau\omega)^{1-\varepsilon} - \tau^{1-\varepsilon})V_N(a_{XS})\} > 0$. This is again positive as: $V_N(a_{XN}) > V_N(a_{XS}) > 0$ (since $a_{XN} > a_{XS}$), and $\omega^{1-\varepsilon} - \tau^{1-\varepsilon} > (\tau\omega)^{1-\varepsilon} - \tau^{1-\varepsilon} > 0$. In sum, we have: $1 > \Delta_1 > \Delta_2 > \Delta_3 > 0$. We further define: $\Delta_d = \rho_1(1-\Delta_1) + (1-\rho_1)(1-\Delta_2) + \frac{1-\rho_2}{2}\frac{E_s}{E_n}(1-\Delta_3) > 0$, which is the denominator in (A.26). We now substitute into (A.26) the expressions for $\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta}$, $\frac{1}{a_D}\frac{da_D}{d\eta}$ and $\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}$ from (A.22) and (A.25). After simplifying, one obtains:

$$\frac{1}{a_I}\frac{da_I}{d\eta} = \frac{1}{\eta}\frac{1}{\Delta_d}\frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\rho_S}{\varepsilon-1}\left[\Delta_3 - \rho_1\Delta_1 - (1-\rho_1)\Delta_2\right] < 0. \tag{A.27}$$

That this last expression is negative follows from the fact that $\rho_1, \rho_2, \Delta_1, \Delta_2, \Delta_3 \in (0,1)$, and that $\Delta_1 > \Delta_2 > \Delta_3$. Moreover, (A.22) and (A.27) imply:

$$\begin{aligned}
\frac{1}{a_I}\frac{da_I}{d\eta} - \frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} &= \frac{1}{\eta}\frac{1}{\Delta_d}\frac{\rho_S}{\varepsilon-1}\left[\frac{E_s}{E_n}\frac{1-\rho_2}{2}(\Delta_3 - 1) + \Delta_d\right] \\
&= \frac{1}{\eta}\frac{1}{\Delta_d}\frac{\rho_S}{\varepsilon-1}\left[\rho_1(1-\Delta_1) + (1-\rho_1)(1-\Delta_2)\right] \\
&> 0.
\end{aligned}$$

Thus, $\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} < \frac{1}{a_I}\frac{da_I}{d\eta} < 0$, which completes the proof of the lemma. ■

Proof of Proposition 1. For part (i), recall from Section 2.2 that: $HOR(a) \equiv A_{sw}\left(a\omega/\alpha\right)^{1-\varepsilon}$, $PLA(a) \equiv A_{ew}\left(\tau a\omega/\alpha\right)^{1-\varepsilon}$, and $RET(a) \equiv A_{ww}\left(\tau a\omega/\alpha\right)^{1-\varepsilon}$. We thus have:

$$\begin{aligned}
\frac{d}{d\eta}\log HOR(a) &= \frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} = -\frac{\rho_S}{\eta}, \\
\frac{d}{d\eta}\log PLA(a) &= \frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} = \frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\rho_S}{\eta}, \text{ and} \\
\frac{d}{d\eta}\log RET(a) &= \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} = \frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\rho_S}{\eta}.
\end{aligned}$$

In the above, we have made use of the fact that: $\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} = \frac{1}{A_{ss}}\frac{dA_{ss}}{d\eta} = -\frac{\rho_S}{\eta}$ from (A.12), and that: $\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} = \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} = \frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\rho_S}{\eta}$, which follows from (A.19), (A.20) and (A.25). Clearly, $\frac{d}{d\eta}\log HOR(a) < 0$, while $\frac{d}{d\eta}\log PLA(a) > 0$ and $\frac{d}{d\eta}\log RET(a) > 0$.

To sign the further cross-derivative effects with respect to ϕ, note that $sign\{\frac{d^2}{d\eta d\phi}\log HOR(a)\} = -sign\{\frac{d\rho_S}{d\phi}\}$. Recall that: $\rho_S \equiv \frac{\left(1-\frac{R\phi+(1-\phi)}{R\phi}\eta\right)\frac{\kappa-\varepsilon+1}{\varepsilon-1}}{1+\left(1-\frac{R\phi+(1-\phi)}{R\phi}\eta\right)\frac{\kappa-\varepsilon+1}{\varepsilon-1}}$, from which it follows that $\frac{d\rho_S}{d\phi} = \frac{\kappa-\varepsilon+1}{\varepsilon-1}\frac{\eta}{R\phi^2}(1-\rho_S)^2 > 0$, so that: $\frac{d^2}{d\eta d\phi}\log HOR(a) < 0$. As for $PLA(a)$ and $RET(a)$, note that the sign of the cross-derivative now depends on: $\frac{d}{d\phi}\frac{1-\rho_2}{2}\rho_S$. From the definition of $\rho_2 = \frac{P_{ss}^{1-\varepsilon}}{P_{ss}^{1-\varepsilon}+2P_{sw}^{1-\varepsilon}}$, we differentiate and simplify to obtain:

$$\begin{aligned}
\frac{d\rho_2}{d\phi} &= \rho_2(1-\rho_2)\left(\frac{1}{P_{ss}^{1-\varepsilon}}\frac{dP_{ss}^{1-\varepsilon}}{d\phi} - \frac{1}{P_{sw}^{1-\varepsilon}}\frac{dP_{sw}^{1-\varepsilon}}{d\phi}\right) \\
&= -(1-\rho_2)\left(\frac{1}{A_{ss}}\frac{A_{ss}}{d\phi} + \frac{1}{P_{sw}^{1-\varepsilon}}\frac{dP_{sw}^{1-\varepsilon}}{d\phi}\right),
\end{aligned} \tag{A.28}$$

where the last step follows from substituting in the expression we get for $\frac{1}{P_{ss}^{1-\varepsilon}}\frac{dP_{ss}^{1-\varepsilon}}{d\phi}$ from log-differentiating the expression for A_{ss} in (A.4), namely:

$$\frac{dA_{ss}}{A_{ss}} = -\rho_2\frac{dP_{ss}^{1-\varepsilon}}{P_{ss}^{1-\varepsilon}} - (1-\rho_2)\frac{dP_{sw}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}}. \tag{A.29}$$

We already have solved for $\frac{1}{A_{ss}}\frac{A_{ss}}{d\phi}$ in (A.14). It remains to derive an expression for $\frac{1}{P_{sw}^{1-\varepsilon}}\frac{dP_{sw}^{1-\varepsilon}}{d\phi}$. For this, we differentiate (A.3) and the price indices in (A.7), (A.8) and (A.9):

$$\frac{dA_{ww}}{A_{ww}} = -\rho_1 \frac{dP_{ww}^{1-\varepsilon}}{P_{ww}^{1-\varepsilon}} - (1-\rho_1)\frac{dP_{ew}^{1-\varepsilon}}{P_{ew}^{1-\varepsilon}}, \tag{A.30}$$

$$\frac{dP_{ww}^{1-\varepsilon}}{P_{ww}^{1-\varepsilon}} = \frac{dN_n}{N_n} + \frac{\kappa-\varepsilon+1}{\varepsilon-1}\left(\Delta_1 \frac{dA_{ww}}{A_{ww}} + (1-\Delta_1)(\varepsilon-1)\frac{da_I}{a_I}\right), \tag{A.31}$$

$$\frac{dP_{ew}^{1-\varepsilon}}{P_{ew}^{1-\varepsilon}} = \frac{dN_n}{N_n} + \frac{\kappa-\varepsilon+1}{\varepsilon-1}\left(\Delta_2 \frac{dA_{ew}}{A_{ew}} + (1-\Delta_2)(\varepsilon-1)\frac{da_I}{a_I}\right), \quad \text{and} \tag{A.32}$$

$$\frac{dP_{sw}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}} = \frac{dN_n}{N_n} + \frac{\kappa-\varepsilon+1}{\varepsilon-1}\left(\Delta_3 \frac{dA_{sw}}{A_{sw}} + (1-\Delta_3)(\varepsilon-1)\frac{da_I}{a_I}\right). \tag{A.33}$$

We moreover have an expression for $\frac{da_I}{a_I}$ that comes from the steps in the derivation of (A.27):

$$\frac{da_I}{a_I} = \frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{1}{\Delta_d}(\rho_1\Delta_1 + (1-\rho_1)\Delta_2 - \Delta_3)\frac{dA_{ss}}{A_{ss}}. \tag{A.34}$$

We now use equations (A.30), (A.31) and (A.32), together with the expression for $\frac{da_I}{a_I}$ from (A.34). After some algebraic manipulations, one obtains:

$$\frac{dN_n}{N_n} = \frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{dA_{ss}}{A_{ss}}(1+\Delta_n), \tag{A.35}$$

we define: $\Delta_n \equiv \frac{\kappa-\varepsilon+1}{\varepsilon-1}\frac{1}{\Delta_d}((\rho_1\Delta_1+(1-\rho_1)\Delta_2)\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\Delta_3)+\Delta_3(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2)))$. Substituting this expression for $\frac{dN_n}{N_n}$ in (A.33), and once again the expression for $\frac{da_I}{a_I}$ from (A.34), we get:

$$\frac{dP_{sw}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}} = \frac{dA_{ss}}{A_{ss}}\left[\frac{E_s}{E_n}\frac{1-\rho_2}{2} + \left(1+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\right)\Delta_n\right]. \tag{A.36}$$

Since $\frac{dA_{ss}}{d\phi} > 0$ from (A.14), it follows from (A.36) that $\frac{1}{P_{sw}^{1-\varepsilon}}\frac{dP_{sw}^{1-\varepsilon}}{d\phi} > 0$, and hence from (A.28) that $\frac{d\rho_2}{d\phi} < 0$. Recall also that $\frac{d\rho_S}{d\phi} > 0$. We therefore have: $sign\{\frac{d^2}{d\eta d\phi}\log PLA(a)\} = sign\{\frac{d^2}{d\eta d\phi}\log RET(a)\} = sign\{\frac{d}{d\phi}\frac{1-\rho_2}{2}\rho_S\} > 0$. This establishes part (i) of the proposition.

For part (ii), we use the expressions for the sales shares by market destination in (2.6)-(2.8). After differentiating these with respect to η and substituting in the expressions: $A_{ww} = A_{ew}$, $\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} = \frac{1}{A_{ss}}\frac{dA_{ss}}{d\eta} = -\frac{\rho_S}{\eta}$, and: $\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} = \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} = \frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\rho_S}{\eta}$, we have:

$$\frac{d}{d\eta}\frac{HOR}{TOT} = -\frac{2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}}{\left(1+2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}\right)^2}\left(1+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\right)\frac{\rho_S}{\eta} < 0, \text{ and}$$

$$\frac{d}{d\eta}\frac{PLA}{TOT} = \frac{d}{d\eta}\frac{RET}{TOT} = \frac{\tau^{\varepsilon-1}\frac{A_{sw}}{A_{ew}}}{\left(2+\tau^{\varepsilon-1}\frac{A_{sw}}{A_{ew}}\right)^2}\left(1+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\right)\frac{\rho_S}{\eta} > 0.$$

We further differentiate the above expressions with respect to ϕ to evaluate the cross-derivatives of interest. Note that:

$$\frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} = -\frac{\rho_S}{\eta}\frac{d}{d\phi}\left\{\frac{2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}}{\left(1+2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}\right)^2}\left(1+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\right)\right\} - \frac{2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}}{\left(1+2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}\right)^2}\left(1+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\right)\frac{d}{d\phi}\left(\frac{\rho_S}{\eta}\right)$$

We have already seen that $\frac{d\rho_S}{d\phi} > 0$. Moreover, when η is sufficiently large – in particular, as it approaches the upper bound of $\frac{R\phi}{R\phi+(1-\phi)}$ implied by the condition $R\frac{\phi}{1-\phi} > \frac{\eta}{1-\eta}$ – one can see from the definition of ρ_S that ρ_S would approach zero. At the same time, it is straightforward to verify from the expressions we have derived for $\frac{d\rho_S}{d\phi}$ and $\frac{d\rho_2}{d\phi}$ that these derivatives will be bounded away from zero, even as ρ_S approaches zero. The first term in the above expression for $\frac{d^2}{d\eta d\phi}\frac{HOR}{TOT}$ would then approach zero, so that the sign of the cross-derivative of interest would be determined by that of the second term and in particular by: $-\frac{d}{d\phi}\left(\frac{\rho_S}{\eta}\right)$. It thus follows that $\frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} < 0$. Applying an analogous argument to $\frac{PLA}{TOT}$ and $\frac{RET}{TOT}$, it follows that for η sufficiently large, $\frac{d^2}{d\eta d\phi}\frac{PLA}{TOT}$ and $\frac{d^2}{d\eta d\phi}\frac{RET}{TOT}$ both inherit the sign of $\frac{d}{d\phi}\left(\frac{\rho_S}{\eta}\right)$. Hence, $\frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} > 0$ and $\frac{d^2}{d\eta d\phi}\frac{RET}{TOT} > 0$. This establishes part (ii) of the proposition.

For part (iii), recall that the aggregate sales outcome measures are given by: $HOR \equiv N_n A_{sw}\left(\omega/\alpha\right)^{1-\varepsilon} V_n(a_I)$, $PLA \equiv N_n A_{ew}\left(\tau\omega/\alpha\right)^{1-\varepsilon} V_n(a_I)$, and $RET \equiv N_n A_{ww}\left(\tau\omega/\alpha\right)^{1-\varepsilon} V_n(a_I)$. Log-differentiating these expressions and substituting in the expressions for $\frac{dN}{d\eta}$ and $\frac{da_I}{d\eta}$ implied by (A.35) and (A.34), we have after some extensive simplification:

$$\begin{aligned}
\frac{d}{d\eta}\log HOR &= \frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta}\left[\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1+\Delta_n)+1+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\kappa-\varepsilon+1}{\varepsilon-1}\frac{1}{\Delta_d}(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\right], \text{ and} \\
\frac{d}{d\eta}\log PLA = \frac{d}{d\eta}\log PLA &= \frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta}\frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\kappa-\varepsilon+1}{\varepsilon-1}\frac{1}{\Delta_d}(\rho_1\Delta_1+(1-\rho_1)(1-\Delta_2))\left(\frac{E_s}{E_n}\frac{1-\rho_2}{2}+1\right)(1-\Delta_3).
\end{aligned}$$

Since $\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} < 0$, this implies: $\frac{d}{d\eta}\log HOR < 0$, $\frac{d}{d\eta}\log PLA < 0$, and $\frac{d}{d\eta}\log RET < 0$. Furthermore, from (A.35), we have:

$$\begin{aligned}
\frac{d}{d\eta}\log N_n &= \frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta}(1+\Delta_n), \text{ and} \\
\frac{d}{d\eta}\log N_n G_n(a_I) &= \frac{d}{d\eta}\log N_n + \kappa\frac{1}{a_I}\frac{da_I}{d\eta} \\
&= \frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta}\frac{E_s}{E_n}\frac{1-\rho_2}{2}\left[1+\Delta_n+\frac{k}{\varepsilon-1}\frac{1}{\Delta_d}(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\right],
\end{aligned}$$

from which it follows that: $\frac{d}{d\eta}\log N_n < 0$ and $\frac{d}{d\eta}\log N_n G_n(a_I) < 0$.

Last but not least, consider the further cross-derivative with respect to ϕ of the above measures of aggregate MNC activity. We illustrate using the case of N_n the proof that this cross-derivative inherits the same sign as the derivative with respect to η when the initial level of η is sufficiently large. (The proofs for $N_n G_n(a_I)$, HOR, PLA, and RET all follow in a similar manner.) Bearing in mind that $\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} = -\frac{\rho_S}{\eta}$, we have:

$$\frac{d^2}{d\eta d\phi}\log N_n = -\frac{\rho_S}{\eta}\frac{d}{d\phi}\left(\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1+\Delta_n)\right) - \frac{E_s}{E_n}\frac{1-\rho_2}{2}(1+\Delta_n)\frac{d}{d\phi}\left(\frac{\rho_S}{\eta}\right)$$

For η sufficiently large, ρ_S approaches zero, and the sign of this cross-derivative will be pinned down by the sign of $-\frac{d}{d\phi}\left(\frac{\rho_S}{\eta}\right)$. It follows that $\frac{d^2}{d\eta d\phi}\log N_n < 0$. This concludes the proof of part (iii) of the proposition.

It is useful to highlight that the condition that η be sufficiently large is not required to sign any of the derivatives with respect to η in parts (i)-(iii), nor is it required to sign the cross-derivative with respect to η and ϕ for the affiliate sales levels in part (i). The condition is only needed to sign the cross-derivatives for the sales shares by destination in part (ii) and for the aggregate sales levels in part (iii). We have also verified that this requirement that η be large is a mild one through numerical examples. As an example, consider the set of parameter values: $R = 1.07$, $\varepsilon = 3.8$, $L_n = L_s = 1$, $f_D = 0.2$, $f_X = 0.15$, $f_S = 0.1$, $f_{En} = f_{Es} = 1$, $\tau = 1.3$, $\omega = 0.7$, $\bar{a}_N = \bar{a}_S = 25$, $\kappa = 4$, $\delta = 0.1$, $\mu = 0.5$, $\phi = 0.5$ and $\eta = 0.1$. The values of the demand elasticity ε and the Pareto shape parameter κ are relatively standard in the literature. The set of parameters moreover respects the ordering of productivity cut-offs in the industry equilibrium ($0 < a_D^{1-\varepsilon} < a_{XN}^{1-\varepsilon} < a_{XS}^{1-\varepsilon} < a_I^{1-\varepsilon}$); specifically, we have: $a_D = 13.53$, $a_{XN} = 11.53$, $a_{XS} = 9.22$ and $a_I = 6.38$. While this already features a low value of η, we nevertheless continue to have all the signs of the cross-derivatives pertaining to parts (ii) and (iii) bearing the predicted signs from the proposition: $\frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} = -0.61$, $\frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} = \frac{d^2}{d\eta d\phi}\frac{RET}{TOT} = 0.30$, $\frac{d^2}{d\eta d\phi}\log N_n = -1.34$, $\frac{d^2}{d\eta d\phi}\log N_n G_n(a_I) = -2.11$, $\frac{d^2}{d\eta d\phi}\log HOR = -2.45$, and $\frac{d^2}{d\eta d\phi}\log PLA = \frac{d^2}{d\eta d\phi}\log RET = -0.50$. ■

Mapping the model to the empirics: Observed private credit and the parameter η. The model counterpart of our empirical measure of private credit over GDP is: $\sum_k N_s^k G^k(a_S^k)\phi^k f_S^k \omega/(\omega L)$, this being the total amount borrowed by domestic firms, summed across all differentiated-varieties sectors k, divided by the total labor income in South. Since f_S, ϕ^k, ω, and L are fixed, it suffices to show that $N_s^k G^k(a_S^k)$, the "number" of successful entrants in the Southern industry in each differentiated-variety sector k, is increasing in η.

We establish this as follows for each such sector, dropping the superscript k for ease of notation. First, log-differentiate the ideal price index, $P_{ss}^{1-\varepsilon}$, given by (A.10):

$$\frac{1}{N_s}\frac{dN_s}{d\eta} = \frac{1}{P_{ss}^{1-\varepsilon}}\frac{dP_{ss}^{1-\varepsilon}}{d\eta} - (\kappa - \varepsilon + 1)\frac{1}{a_S}\frac{da_S}{d\eta}. \tag{A.37}$$

We therefore have: $\frac{d}{d\eta}\log N_s G_s(a_S) = \frac{1}{N_s}\frac{dN_S}{d\eta} + \frac{G_s'(a_S)a_S}{G_s(a_S)}\frac{1}{a_S}\frac{da_S}{d\eta} = \frac{1}{N_s}\frac{dN_S}{d\eta} + \kappa\frac{1}{a_S}\frac{da_S}{d\eta} = \frac{1}{P_{ss}^{1-\varepsilon}}\frac{dP_{ss}^{1-\varepsilon}}{d\eta} + (\varepsilon-1)\frac{1}{a_S}\frac{da_S}{d\eta}$, where we have made use of (A.37) to obtain the last expression. We have seen from Lemma 1 that $\frac{da_S}{d\eta} > 0$. As $\varepsilon > 1$, it will thus suffice to show that $\frac{1}{P_{ss}^{1-\varepsilon}}\frac{dP_{ss}^{1-\varepsilon}}{d\eta} > 0$, in order to conclude that $\frac{d}{d\eta}\log N_s G_s(a_S) > 0$.

For this, we log-differentiate (A.4) to obtain: $\frac{dA_{sw}}{A_{sw}} = -\rho_2\frac{dP_{ss}^{1-\varepsilon}}{P_{ss}^{1-\varepsilon}} - (1-\rho_2)\frac{dP_{sw}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}}$. Substituting in the expression for $\frac{dA_{sw}}{A_{sw}}$ from (A.21) into this last equation, and rearranging, gives:

$$\rho_2\frac{1}{P_{ss}^{1-\varepsilon}}\frac{dP_{ss}^{1-\varepsilon}}{d\eta} = -(\varepsilon-1)\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} - (1-\rho_2)\frac{1}{P_{sw}^{1-\varepsilon}}\frac{dP_{sw}^{1-\varepsilon}}{d\eta}. \tag{A.38}$$

Now, log-differentiating (A.9) yields:

$$\frac{1}{P_{sw}^{1-\varepsilon}}\frac{dP_{sw}^{1-\varepsilon}}{d\eta} = \frac{1}{N_n}\frac{dN_n}{d\eta} + (\kappa - \varepsilon + 1)\left(\Delta_3\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} + (1-\Delta_3)\frac{1}{a_I}\frac{da_I}{d\eta}\right).$$

Since $\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} < 0$ and $\frac{1}{a_I}\frac{da_I}{d\eta} < 0$ from Lemma 2, and $\frac{1}{N_n}\frac{dN_n}{d\eta} < 0$ from Proposition 1, it follows that: $\frac{1}{P_{sw}^{1-\varepsilon}}\frac{dP_{sw}^{1-\varepsilon}}{d\eta} < 0$. From (A.38), we immediately have: $\frac{1}{P_{ss}^{1-\varepsilon}}\frac{dP_{ss}^{1-\varepsilon}}{d\eta} > 0$, so that $\frac{d}{d\eta}\log N_s G_s(a_S) > 0$. We can thus conclude that total private credit extended in South is increasing with η. ■

Proof of Lemma 3. First, observe that the equilibrium for South's differentiated-varieties sector is still determined by (2.5) and (A.6) as in the baseline model. Thus, Lemma 1 holds and the expressions for $\frac{da_S}{d\eta}$ and $\frac{dA_{ss}}{d\eta}$ from (A.11) and (A.12) still apply. As for the Western industry, only two equations in the equilibrium system are altered relative to the baseline model. The first of these is the FDI cut-off which is now given by:

$$\tilde{a}_I^{1-\varepsilon} = \frac{R\phi}{\eta}a_I^{1-\varepsilon}, \tag{A.39}$$

The second is the free-entry condition as the costs of affiliate borrowing from host-country sources must now be reflected in the fixed costs of firms that undertake FDI:

$$\begin{aligned} \delta f_{En} \quad = \quad & (1-\alpha)A_{ww}\left(\frac{1}{\alpha}\right)^{1-\varepsilon}(V_n(a_D) - V_n(\tilde{a}_I)) - f_D(G_n(a_D) - G_n(\tilde{a}_I)) \\ & +(1-\alpha)A_{ew}\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XN}) - V_n(\tilde{a}_I)) - f_X(G_n(a_{XN}) - G_n(\tilde{a}_I)) \\ & +(1-\alpha)A_{sw}\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XS}) - V_n(\tilde{a}_I)) - f_X(G_n(a_{XS}) - G_n(\tilde{a}_I)) \\ & +(1-\alpha)\left(A_{ww}\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} + A_{ew}\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} + A_{sw}\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}\right)V_n(\tilde{a}_I) \\ & -(f_D + 2f_X + (R\phi + (1-\phi))(f_I - f_D))G_n(\tilde{a}_I). \end{aligned} \tag{A.40}$$

Differentiating equation (A.39) yields:

$$(\varepsilon-1)\frac{d\tilde{a}_I}{\tilde{a}_I} = \frac{1}{\Delta_d}\left[\rho_1(1-\Delta_1)\frac{dA_{ww}}{A_{ww}} + (1-\rho_1)(1-\Delta_2)\frac{dA_{ew}}{A_{ew}} + \frac{1-\rho_2}{2}\frac{E_s}{E_n}(1-\Delta_3)\frac{dA_{sw}}{A_{sw}}\right] + \frac{d\eta}{\eta} - \frac{d\phi}{\phi}. \quad \text{(A.41)}$$

The additional terms involving $\frac{d\eta}{\eta}$ and $\frac{d\phi}{\phi}$ on the right-hand side capture the direct effect that Southern financing conditions have on Western firms. As for (A.40), differentiating this yields an equation analogous to (A.23), where a similar set of manipulations as from the proof of Lemma 2 leads to:

$$\begin{aligned}
0 =& \left[(1-\alpha)A_{ww}\left(\left(\frac{1}{\alpha}\right)^{1-\varepsilon}V_n(a_D) + \left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right)V_n(\tilde{a}_I)\right)\right]\frac{dA_{ww}}{A_{ww}} \\
&+ \left[(1-\alpha)A_{ew}\left(\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}V_n(a_{XN}) + \left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)V_n(\tilde{a}_I)\right)\right]\frac{dA_{ew}}{A_{ew}} \\
&+ \left[(1-\alpha)A_{sw}\left(\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}V_n(a_{XS}) + \left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)V_n(\tilde{a}_I)\right)\right]\frac{dA_{sw}}{A_{sw}} \\
&+ \left[(1-\alpha)\left(A_{ww}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right) + A_{ew}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)\right.\right. \\
&\left.\left. + A_{sw}\left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)\right)V_n'(\tilde{a}_I) - (R\phi + (1-\phi))(f_I - f_D)G_n'(\tilde{a}_I)\right]d\tilde{a}_I \\
&- (R-1)(f_I - f_D)G_n(\tilde{a}_I)d\phi. \qquad \text{(A.42)}
\end{aligned}$$

To further simplify this, we now need to bear in mind that the coefficient of the term in $d\tilde{a}_I$ is no longer equal to 0. This is because:

$$\begin{aligned}
&(1-\alpha)\left[A_{ww}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right) + A_{ew}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right) + A_{sw}\left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)\right]V_n'(\tilde{a}_I) \\
&\quad -(R\phi + (1-\phi))(f_I - f_D)G_n'(\tilde{a}_I) \\
=& (1-\alpha)\left(1 - \frac{R\phi + (1-\phi)}{R\phi}\eta\right)\left[A_{ww}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right) + A_{ew}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right) + A_{sw}\left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}\right)\right]V_n'(\tilde{a}_I),
\end{aligned}$$

where the last step follows from using the definition of $\tilde{a}_I^{1-\varepsilon}$ from (A.39) to substitute out for $(f_I - f_D)$, as well as from using Leibniz's rule to replace $G_n'(\tilde{a}_I)$ with $\tilde{a}_I^{\varepsilon-1}V_n'(\tilde{a}_I)$. We now follow analogous algebraic steps as in the proof of Lemma 2, in particular, substituting in the definitions of the price indices (A.7)-(A.9), as well as the definitions of ρ_1 and Δ_d. This allows us to rewrite (A.42) as:

$$\frac{1}{\Delta_d}\left[\rho_1\frac{dA_{ww}}{A_{ww}} + (1-\rho_1)\frac{dA_{ew}}{A_{ew}} + \frac{1-\rho_2}{2}\frac{E_s}{E_n}\frac{dA_{sw}}{A_{sw}}\right] + \frac{\rho_S}{1-\rho_S}(\varepsilon-1)\frac{d\tilde{a}_I}{\tilde{a}_I} - \frac{R-1}{R}\tilde{a}_I^{1-\varepsilon}\eta\frac{G_n(\tilde{a}_I)}{V_n(\tilde{a}_I)}\frac{d\phi}{\phi} = 0.$$

Since $\tilde{a}_I^{1-\varepsilon}\frac{G_n(\tilde{a}_I)}{V_n(\tilde{a}_I)} = \frac{k-\varepsilon+1}{k}$ under the Pareto distribution, the above simplifies further to:

$$\frac{1}{\Delta_d}\left[\rho_1\frac{dA_{ww}}{A_{ww}} + (1-\rho_1)\frac{dA_{ew}}{A_{ew}} + \frac{1-\rho_2}{2}\frac{E_s}{E_n}\frac{dA_{sw}}{A_{sw}}\right] + \frac{\rho_S}{1-\rho_S}(\varepsilon-1)\frac{d\tilde{a}_I}{\tilde{a}_I} - \frac{R-1}{R}\eta\frac{k-\varepsilon+1}{k}\frac{d\phi}{\phi} = 0. \quad \text{(A.43)}$$

Solving (A.41) and (A.43) simultaneously and after some tedious algebra, we obtain:

$$\frac{dA_{ww}}{A_{ww}} = \frac{1}{1-\tilde{\rho}_S}\left[-\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S\Delta_3)\frac{dA_{sw}}{A_{sw}} - \Delta_d\rho_S\frac{d\eta}{\eta} + \Delta_d\left(\rho_S + \frac{R-1}{R}\eta\frac{k-\varepsilon+1}{k}(1-\rho_S)\right)\frac{d\phi}{\phi}\right], \text{ and} \quad \text{(A.44)}$$

$$(\varepsilon-1)\frac{d\tilde{a}_I}{\tilde{a}_I} = \frac{1-\rho_S}{1-\tilde{\rho}_S}\left[\frac{1}{\Delta_d}(\rho_1\Delta_1 + (1-\rho_1)\Delta_2 - \Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{dA_{sw}}{A_{sw}} + \frac{d\eta}{\eta} + \left(-1 + \frac{R-1}{R}\eta\frac{k-\varepsilon+1}{k}\right)\frac{d\phi}{\phi}\right], \quad \text{(A.45)}$$

where we define: $\tilde{\rho}_S \equiv \rho_S(\rho_1\Delta_1 + (1-\rho_1)\Delta_2)$. Note that $\tilde{\rho}_S \in (0,1)$, since $\rho_S \in (0,1)$ and $0 < \rho_1\Delta_1 + (1-\rho_1)\Delta_2 < \rho_1 + (1-\rho_1) < 1$.

Bear in mind that we continue to have $(\varepsilon-1)\frac{da_{XS}}{a_{XS}} = \frac{dA_{sw}}{A_{sw}} = \frac{dA_{ss}}{A_{ss}}$. Since $\frac{1}{A_{ss}}\frac{dA_{ss}}{d\eta} = -\frac{\rho_S}{\eta}$ from the industry equilibrium in South, (A.45) implies that:

$$(\varepsilon-1)\frac{1}{\tilde{a}_I}\frac{d\tilde{a}_I}{d\eta} = \frac{1-\rho_S}{1-\tilde{\rho}_S}\frac{1}{\eta}\left[-\rho_S\frac{1}{\Delta_d}(\rho_1\Delta_1 + (1-\rho_1)\Delta_2 - \Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2} + 1\right].$$

Note that: (i) $\rho_1\Delta_1 + (1-\rho_1)\Delta_2 - \Delta_3 > 0$, since $\Delta_1, \Delta_2 > \Delta_3$; and (ii) $\frac{E_s}{E_n}\frac{1-\rho_2}{2}(\rho_1\Delta_1 + (1-\rho_1)\Delta_2 - \Delta_3) < \frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\Delta_3) < \Delta_d$, since $\Delta_1, \Delta_2 < 1$. These two facts imply that: $\frac{E_s}{E_n}\frac{1-\rho_2}{2}\frac{\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3}{\Delta_d} \in (0,1)$. Since we also have $\rho_S \in (0,1)$, it follows from the above expression that $\frac{1}{\tilde{a}_I}\frac{d\tilde{a}_I}{d\eta} > 0$, as claimed in part (i) of Lemma 3. We have also already seen that: $\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} = -\frac{1}{\eta}\frac{\rho_S}{\varepsilon-1} < 0$, which is part (ii) of the lemma.

As for part (iii), we make use of (A.44) and the fact that $(\varepsilon-1)\frac{da_D}{a_D} = (\varepsilon-1)\frac{da_{XN}}{a_{XN}} = \frac{dA_{ww}}{A_{ww}} = \frac{dA_{ew}}{A_{ew}}$. Moreover, since $\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} = -\frac{\rho_S}{\eta}$, we obtain:

$$(\varepsilon-1)\frac{1}{a_D}\frac{da_D}{d\eta} = \frac{\rho_S}{1-\tilde{\rho}_S}\frac{1}{\eta}\left[\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S)\Delta_3 - \rho_1(1-\Delta_1) - (1-\rho_1)(1-\Delta_2)\right]. \qquad \text{(A.46)}$$

It follows that:

$$\frac{1}{a_D}\frac{da_D}{d\eta} - \frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} = \frac{1-\rho_S}{1-\tilde{\rho}_S}\frac{\rho_S}{\varepsilon-1}\frac{1}{\eta}\left[\frac{E_s}{E_n}\frac{1-\rho_2}{2}\Delta_3 + \rho_1\Delta_1 + (1-\rho_1)\Delta_2\right] > 0.$$

This establishes part (iii) of Lemma 3. As for parts (iv) and (v) of the lemma, these follow immediately from applying (A.19)-(A.21). ■

Proof of Proposition 2. We start first with the outcome measures of affiliate sales levels by market destination. As in the proof of Proposition 1, we have:

$$\frac{d}{d\eta}\log HOR(a) = \frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} = -\frac{\rho_S}{\eta} < 0.$$

As in that preceding proof, we also saw that $\frac{d\rho_S}{d\phi} > 0$, which implies that $\frac{d^2}{d\eta d\phi}\log HOR(a) < 0$. In the case with host-country borrowing, we now have:

$$\begin{aligned}\frac{d}{d\eta}\log PLA(a) &= \frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} \\ &= \frac{\rho_S}{1-\tilde{\rho}_S}\frac{1}{\eta}\left[\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S)\Delta_3 - \rho_1(1-\Delta_1) - (1-\rho_1)(1-\Delta_2)\right],\end{aligned}$$

where the last step follows from (A.46). From the above, we can see that $\frac{d}{d\eta}\log PLA(a)$ (and hence $\frac{d}{d\eta}\log RET(a)$) cannot be conclusively signed. Note further that the sign of $\frac{d^2}{d\eta d\phi}\log PLA(a)$ depends on the sign of:

$$\begin{aligned}&\frac{1}{1-\tilde{\rho}_S}\frac{1}{\eta}\left[\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S)\Delta_3 - \rho_1(1-\Delta_1) - (1-\rho_1)(1-\Delta_2)\right]\frac{d\rho_S}{d\phi}\ldots \\ &+\rho_S\frac{d}{d\phi}\frac{1}{1-\tilde{\rho}_S}\frac{1}{\eta}\left[\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S)\Delta_3 - \rho_1(1-\Delta_1) - (1-\rho_1)(1-\Delta_2)\right]\end{aligned}$$

For η sufficiently large such that ρ_S is close to zero, and given that $\frac{d\rho_S}{d\phi} > 0$, we have that $\frac{d^2}{d\eta d\phi}\log PLA(a)$ (and hence $\frac{d^2}{d\eta d\phi}\log RET(a)$ too) inherits the sign of $\frac{d}{d\eta}\log PLA(a)$. This establishes part (i) of the proposition.

For the sales shares by destination, one can show that:

$$\frac{d}{d\eta}\frac{HOR}{TOT} = -\frac{2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}}{\left(1+2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}\right)^2}\frac{\rho_S}{\eta}\frac{1-\rho_S}{1-\tilde{\rho}_S}\left(\rho_1\Delta_1+(1-\rho_1)\Delta_2+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\Delta_3\right), \text{ and}$$

$$\frac{d}{d\eta}\frac{PLA}{TOT} = \frac{d}{d\eta}\frac{RET}{TOT} = \frac{\tau^{\varepsilon-1}\frac{A_{sw}}{A_{ew}}}{\left(2+\tau^{\varepsilon-1}\frac{A_{sw}}{A_{ew}}\right)^2}\frac{\rho_S}{\eta}\frac{1-\rho_S}{1-\tilde{\rho}_S}\left(\rho_1\Delta_1+(1-\rho_1)\Delta_2+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\Delta_3\right).$$

From these expressions, we immediately have: $\frac{d}{d\eta}\frac{HOR}{TOT} < 0$, $\frac{d}{d\eta}\frac{PLA}{TOT} > 0$ and $\frac{d}{d\eta}\frac{RET}{TOT} > 0$. We show how to further sign the cross-derivative of interest for the case of $\frac{HOR}{TOT}$. Differentiating the above expression with respect to ϕ yields:

$$\begin{aligned}\frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} &= -\frac{\rho_S}{\eta}\frac{d}{d\phi}\left\{\frac{2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}}{\left(1+2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}\right)^2}\frac{1-\rho_S}{1-\tilde{\rho}_S}\left(\rho_1\Delta_1+(1-\rho_1)\Delta_2+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\Delta_3\right)\right\}\\ &\quad -\frac{2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}}{\left(1+2\tau^{1-\varepsilon}\frac{A_{ew}}{A_{sw}}\right)^2}\frac{1-\rho_S}{1-\tilde{\rho}_S}\left(\rho_1\Delta_1+(1-\rho_1)\Delta_2+\frac{E_s}{E_n}\frac{1-\rho_2}{2}\Delta_3\right)\frac{d}{d\phi}\left(\frac{\rho_S}{\eta}\right).\end{aligned}$$

For η sufficiently large, ρ_S approaches zero and the sign of this cross-derivative will be pinned down by the sign of the term in the second row above. As $\frac{d\rho_S}{d\phi} > 0$, we have: $\frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} < 0$. By differentiating the expression for $\frac{d}{d\eta}\frac{PLA}{TOT}$ above with respect to ϕ and applying a similar argument, one can show too that $\frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} > 0$ and $\frac{d^2}{d\eta d\phi}\frac{RET}{TOT} > 0$. This establishes part (ii) of the proposition.

For part (iii), we first work out expressions for the derivatives of the aggregate variables of interest. Observe that the expressions for the log-derivatives of A_{ww}, $P_{ww}^{1-\varepsilon}$ and $P_{ew}^{1-\varepsilon}$ in equations (A.30)-(A.32) remain valid in the model with host-country financing. Eliminating $\frac{dP_{ww}^{1-\varepsilon}}{P_{ww}^{1-\varepsilon}}$ and $\frac{dP_{ew}^{1-\varepsilon}}{P_{ew}^{1-\varepsilon}}$ from these equations and using (A.19), we have:

$$\begin{aligned}\frac{1}{N_n}\frac{dN_n}{d\eta} &= \frac{1}{1-\tilde{\rho}_S}\frac{1}{\eta}\times\Bigg[-\rho_S\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S\Delta_3)\left(1+\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1\Delta_1+(1-\rho_1)\Delta_2)\right)\\ &\quad+\rho_S\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\frac{1}{\Delta_d}(1-\rho_S)(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2}\\ &\quad+\rho_S\left(1+\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1\Delta_1+(1-\rho_1)\Delta_2)\right)\Delta_d\\ &\quad-(1-\rho_S)\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\Bigg].\end{aligned} \tag{A.47}$$

In turn, how the number of multinationals, $N_nG_n(\tilde{a}_I)$, responds to η is given by: $\frac{d}{d\eta}\log N_nG_n(\tilde{a}_I) = \frac{1}{N_n}\frac{dN_n}{d\eta} + \frac{G_n'(\tilde{a}_I)\tilde{a}_I}{G_n(\tilde{a}_I)}\frac{1}{\tilde{a}_I}\frac{d\tilde{a}_I}{d\eta} = \frac{1}{N_n}\frac{dN_n}{d\eta} + \kappa\frac{1}{\tilde{a}_I}\frac{d\tilde{a}_I}{d\eta}$, where $\frac{G_n'(a)a}{G_n(a)} = \kappa$ for the Pareto distribution. Using (A.47), this yields:

$$\begin{aligned}\frac{1}{N_n}\frac{dN_n}{d\eta}+\kappa\frac{1}{\tilde{a}_I}\frac{d\tilde{a}_I}{d\eta} &= \frac{1}{1-\tilde{\rho}_S}\frac{1}{\eta}\times\Bigg[-\rho_S\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S\Delta_3)\left(1+\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1\Delta_1+(1-\rho_1)\Delta_2)\right)\\ &\quad+\rho_S\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\frac{1}{\Delta_d}(1-\rho_S)(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2}\\ &\quad-\rho_S\frac{k}{\varepsilon-1}\frac{1}{\Delta_d}(1-\rho_S)(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2}\\ &\quad+\rho_S\left(1+\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1\Delta_1+(1-\rho_1)\Delta_2)\right)\Delta_d\\ &\quad-\frac{k-\varepsilon+1}{\varepsilon-1}(1-\rho_S)(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))+\frac{k}{\varepsilon-1}(1-\rho_S)\Bigg].\end{aligned} \tag{A.48}$$

When ρ_S approaches zero, the sign of (A.48) is pinned down by:

$$(1-\rho_S)\left[-\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))+\frac{k}{\varepsilon-1}\right] > 0.$$

As for the effect on aggregate horizontal sales, after some extensive substitution and simplification, we have:

$$\begin{aligned}
\frac{d}{d\eta}\ln HOR &= \frac{1}{N_n}\frac{dN_n}{d\eta}+\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta}+(\kappa-\varepsilon+1)\frac{1}{\tilde{a}_I}\frac{d\tilde{a}_I}{d\eta}\\
&= \frac{1}{1-\tilde{\rho}_S}\frac{1}{\eta}\times\Bigg[-\rho_S\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S\Delta_3)\left(1+\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1\Delta_1+(1-\rho_1)\Delta_2)\right)\\
&\quad+\rho_S\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\frac{1}{\Delta_d}(1-\rho_S)(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2}\\
&\quad-\rho_S(1-\rho_S(\rho_1\Delta_1+(1-\rho_1)\Delta_2))\\
&\quad-\rho_S\frac{k-\varepsilon+1}{\varepsilon-1}\frac{1}{\Delta_d}(1-\rho_S)(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2}\\
&\quad+\rho_S\left(1+\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1\Delta_1+(1-\rho_1)\Delta_2)\right)\Delta_d\\
&\quad+\frac{k-\varepsilon+1}{\varepsilon-1}(1-\rho_S)\left(1-\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2)\right)\Bigg]. \qquad (A.49)
\end{aligned}$$

The sign of $\frac{d}{d\eta}\ln HOR$ as ρ_S approaches zero is thus pinned down by:

$$\frac{k-\varepsilon+1}{\varepsilon-1}(1-\rho_S)\left(1-\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2)\right) \quad > \quad 0.$$

Likewise, for aggregate platform and return sales, we have:

$$\begin{aligned}
\frac{d}{d\eta}\ln PLA = \frac{d}{d\eta}\ln RET &= \frac{1}{N_n}\frac{dN_n}{d\eta}+(\varepsilon-1)\frac{1}{a_D}\frac{da_D}{d\eta}+(\kappa-\varepsilon+1)\frac{1}{\tilde{a}_I}\frac{d\tilde{a}_I}{d\eta}\\
&= \frac{1}{1-\tilde{\rho}_S}\frac{1}{\eta}\times\Bigg[-\rho_S\frac{E_s}{E_n}\frac{1-\rho_2}{2}(1-\rho_S\Delta_3)\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1\Delta_1+(1-\rho_1)\Delta_2)\\
&\quad+\rho_S\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\frac{1}{\Delta_d}(1-\rho_S)(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2}\\
&\quad-\rho_S\frac{k-\varepsilon+1}{\varepsilon-1}\frac{1}{\Delta_d}(1-\rho_S)(\rho_1\Delta_1+(1-\rho_1)\Delta_2-\Delta_3)\frac{E_s}{E_n}\frac{1-\rho_2}{2}\\
&\quad+\rho_S\frac{k-\varepsilon+1}{\varepsilon-1}(\rho_1\Delta_1+(1-\rho_1)\Delta_2)\Delta_d\\
&\quad+\frac{k-\varepsilon+1}{\varepsilon-1}(1-\rho_S)\left(1-\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2)\right)\Bigg]. \qquad (A.50)
\end{aligned}$$

The sign of $\frac{d}{d\eta}\ln PLA$ and $\frac{d}{d\eta}\ln RET$ as ρ_S approaches zero is thus pinned down by:

$$\frac{k-\varepsilon+1}{\varepsilon-1}(1-\rho_S)\left(1-\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2)\right) \quad > \quad 0.$$

This concludes the proof for part (iii) of the proposition.

We provide below an illustration that the condition that η be sufficiently large is relatively mild in practice. Consider the following set of parameters: $R = 1.07$, $\varepsilon = 3.8$, $L_n = L_s = 1$, $f_D = 0.2$, $f_X = 0.15$, $f_S = 0.1$, $f_{En} = f_{Es} = 1$, $\tau = 1.3$, $\omega = 0.5$, $\bar{a}_N = \bar{a}_S = 25$, $\kappa = 4$, $\delta = 0.1$, $\mu = 0.5$, $\phi = 0.5$ and $\eta = 0.1$. This is the same set as was considered in the numerical example at the end of the proof of Proposition 1, except for $\omega = 0.5$; it turns out to be convenient for our illustration to consider a lower value of the Southern wage here.[4] Despite the low value of $\eta = 0.1$, we nevertheless have: $\frac{d}{d\eta}\frac{HOR}{TOT} = -0.15$, $\frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} = -0.14$, $\frac{d}{d\eta}\frac{PLA}{TOT} = \frac{d}{d\eta}\frac{RET}{TOT} = 0.08$, $\frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} = \frac{d^2}{d\eta d\phi}\frac{RET}{TOT} = 0.07$, $\frac{d}{d\eta}\log N_n G_n(\tilde{a}_I) = 11.73$, $\frac{d}{d\eta}\log HOR = 0.91$, and $\frac{d}{d\eta}\log PLA = \frac{d}{d\eta}RET = 1.95$, in accord with Proposition 2. Note too that with these parameters, we further have: $\frac{d^2}{d\eta d\phi}\log N_n G_n(\tilde{a}_I) = 0.62$, $\frac{d^2}{d\eta d\phi}\log HOR = 0.01$, and $\frac{d^2}{d\eta d\phi}\log PLA = \frac{d^2}{d\eta d\phi}RET = 0.83$. Although we could not derive an analytical result related to the signs of the cross-derivatives for these aggregate outcome measures of MNC activity, the above

[4] We continue to have the productivity cut-offs in West satisfying: $0 < a_D^{1-\varepsilon} < a_{XN}^{1-\varepsilon} < a_{XS}^{1-\varepsilon} < a_I^{1-\varepsilon}$. Specifically, we have: $a_D = 11.89$, $a_{XN} = 10.14$, $a_{XS} = 5.84$ and $a_I = 4.48$.

example nevertheless confirms that there exist parameter values such that the cross-derivatives of interest also inherit the sign of the derivative of the respective outcome variable with respect to η.

If we were moreover to lower the initial value of η to 0.05 (holding the other parameter values constant), this would in fact yield a parametrization in which the cross-derivatives of both the affiliate-level and aggregate measures of MNC activity would be consistent with what we report in our main empirical results in Table 4 of the paper.[5] In particular, we have for affiliate-level sales: $\frac{d^2}{d\eta d\phi}\log HOR(a) = -0.83$, and $\frac{d^2}{d\eta d\phi}\log PLA(a) = \frac{d^2}{d\eta d\phi}\log RET(a) = 0.70$; for the sales shares by destination market: $\frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} = -0.31$, and $\frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} = \frac{d^2}{d\eta d\phi}\frac{RET}{TOT} = 0.15$; and for the aggregate levels of MNC activity: $\frac{d^2}{d\eta d\phi}\log N_n G_n(\tilde{a}_I) = 1.15$, $\frac{d^2}{d\eta d\phi}\log HOR = 0.08$, and $\frac{d^2}{d\eta d\phi}\log PLA = \frac{d^2}{d\eta d\phi}\log RET = 1.61$. ■

B Four Model Extensions

We briefly explore four extensions of the model. These allow us to assess the robustness of the competition and financing effects, when plausible modifications are made to certain key features of the setup. The extensions respectively relate to incorporating: (i) home-bias in consumption; (ii) exports by host-country firms in South; (iii) endogenous host-country wages; and (iv) multiple FDI host countries. We briefly describe each of these extensions below, and provide a more detailed treatment in Section C of this Online Appendix. Note that we develop these extensions focusing on the level effect of host-country financial development, η, in order to simplify the exposition; where applicable, we illustrate the results related to the further cross-derivatives with respect to sector external finance dependence ϕ through computational examples.

B.1 Home-bias in consumption

In the model presented in the main paper, platform and return sales respond identically to host-country financial development, even though this does not hold strictly in the data. While there are various ways to relax this from a modeling perspective, one approach that is analytically tractable (and which preserves much of the underlying symmetry in our framework) is to introduce home-bias in consumer preferences. Specifically, assume the utility functions in each country ($n = w, e$) are now:

$$U_n = (y_n)^{\mu^0} \prod_{k=1}^{K} \left(\sum_{j\in\{e,w\}} \left(\int_{\Omega_{nj}^k} x_{nj}^k(a)^{\alpha^k}\, dG_j^k(a) \right)^{\frac{\beta^k}{\alpha^k}} \right)^{\frac{\mu^k}{\beta^k}}, \text{ and}$$

$$U_s = (y_s)^{\mu^0} \prod_{k=1}^{K} \left(\sum_{j\in\{e,w,s\}} \left(\int_{\Omega_{sj}^k} x_{sj}^k(a)^{\alpha^k}\, dG_j^k(a) \right)^{\frac{\beta^k}{\alpha^k}} \right)^{\frac{\mu^k}{\beta^k}}.$$

In contrast to (A.1) and (A.2), the sub-utility derived from differentiated-varieties is now a two-tiered CES function. We assume that the elasticity of substitution for varieties from the same country exceeds the elasticity of substitution for varieties from different countries, i.e., $\varepsilon^k = \frac{1}{1-\alpha^k} > \theta^k = \frac{1}{1-\beta^k} > 1$. This translates into home bias, as varieties are closer substitutes if they bear the same country-of-origin. (This identity of the variety

[5] The initial industry equilibrium in West now features: $a_D = 12.35$, $a_{XN} = 10.53$, $a_{XS} = 6.24$ and $a_I = 3.65$.

travels with the firm regardless of the location where the variety is produced, through its product design and attributes.)

Under this richer utility specification, an improvement in Southern financial development once again spurs entry by domestic firms and increases competition for Western varieties in each differentiated varieties sector k. However, for each such sector (omitting the superscript k), we can show that demand for Western products now increases proportionally more in East than in West ($\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} > \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} > 0$), while the $a_{XN}^{1-\varepsilon}$ cut-off falls proportionally more than the $a_D^{1-\varepsilon}$ cut-off ($\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta} > \frac{1}{a_D}\frac{da_D}{d\eta} > 0$). In the detailed derivations in Section C.1, we prove that Proposition 1 remains true in its entirety. However, a further prediction can now be added:

Proposition 3 *With home bias in consumer preferences, (i)* $\frac{d}{d\eta}PLA(a) > \frac{d}{d\eta}RET(a)$*; (ii)* $\frac{d}{d\eta}\frac{PLA(a)}{TOT(a)} = \frac{d}{d\eta}\frac{PLA}{TOT} > \frac{d}{d\eta}\frac{RET(a)}{TOT(a)} = \frac{d}{d\eta}\frac{RET}{TOT}$*; and (iii)* $\frac{d}{d\eta}PLA > \frac{d}{d\eta}RET$*.*

The increase in multinational affiliates' export-platform sales now exceeds that of their return sales to West. Intuitively, a Western MNC faces tougher competition in its own home market than in East. This occurs because other Western varieties are closer substitutes in consumption than Eastern varieties, and a margin of Western firms (with productivity $a_D^{1-\varepsilon} < a^{1-\varepsilon} < a_{XN}^{1-\varepsilon}$) sell only at home but not in East.

B.2 Southern exports

We next extend the model to allow Western and Eastern consumers to demand Southern varieties. Southern firms can now exert competitive pressure on Western and Eastern manufacturers not only in South, but also in their respective home markets. Below, we briefly sketch how we incorporate Southern exporting, and discuss how this qualifies some of the previous predictions; a detailed exposition is in Section C.2.

Assume that Southern firms can export by incurring the iceberg trade cost, $\tau > 1$, as well as an upfront fixed cost of $f_{X,ws}$ units of Southern labor to serve each of the markets West and East. Southern firms that export require external finance for $f_{X,ws}$, and face credit constraints in raising this capital just as they do for their domestic operations. Financial development in South thus increases domestic firm entry, and also enables more Southern firms to export. This raises competition in the goods markets in all three countries, but to different degrees. Because the equilibrium in South's differentiated-varieties sector now includes a feedback effect from demand in West and East, we analyze this case through computational examples. We build these examples to examine values of $f_{X,ws}$ that lie between the fixed cost of exporting for Western firms (f_X) and the fixed cost of FDI (f_I). In general, we continue to find that improving financial institutions in South continues to increase competition in that market, so that affiliates decrease their share of sales to the local economy, while raising the shares sold to West/East. At the same time, it is possible that the levels of an individual affiliate's sales to all three markets decrease, reflecting the feature that Southern firms now pose more intense competition to firms from West/East in all three markets, in particular in the Western/Eastern markets through their ability to export there.

B.3 Endogenous host-country wages

Up to this point, we have made the assumption that the host-country wage, ω, is pinned down exogenously by the marginal product of labor in the homogeneous-good sector. This facilitated the analytical tractability of the model, allowing us to highlight the effects of interest without considering feedback effects from the host-country labor market. We now consider the implications of relaxing this assumption.

To do so, consider the special case of the baseline model in which $\mu = 1$ in the utility functions of both Northern and Southern consumers, (A.1) and (A.2). To keep the exploration as clean as possible, we also focus on the case in which $K = 1$, namely where there is exactly one differentiated-varieties sector. We continue to adopt the wage in North as the numeraire. Setting $\mu = 1$ effectively shuts down the homogenous-good sector, so that the Southern wage ω is now determined endogenously by a labor market clearing condition in South:

$$\begin{aligned} L_s &= N_s A_{ss} \left(\frac{\omega}{\alpha}\right)^{-\varepsilon} V_s(a_S) + N_s R f_S G_s(a_S) + \delta N_s f_{Es} \\ &\quad + 2N_n \left[A_{ww} \tau \left(\frac{\tau\omega}{\alpha}\right)^{-\varepsilon} + A_{ew} \tau \left(\frac{\tau\omega}{\alpha}\right)^{-\varepsilon} + A_{sw} \left(\frac{\omega}{\alpha}\right)^{-\varepsilon} \right] V_n(a_I). \end{aligned} \tag{B.1}$$

This equates the Southern labor supply to the total use of labor in that economy. Note that the expression on the right-hand side of the first line of (C.54) corresponds to the use of Southern labor by Southern firms, including the labor that is used to service the domestic fixed cost of production and domestic entry. (In particular, the number of Southern entrants in each period is equal to the number who exit exogenously, i.e., δN_s, in order for the number of Southern firms to be constant in the steady state.) The second line of (C.54) in turn corresponds to the use of Southern labor by the multinational affiliates of Western and Eastern firms. (As an implication of Walras' Law, it is straightforward to show that the labor market clearing condition for North is redundant in the system of equations that defines the model equilibrium.)

With Southern wages now adjusting endogenously, an improvement in host-country financial development that spurs more Southern entry will raise the overall demand for labor in South and thus lead to a rise in ω. Intuitively, the rise in Southern incomes can now dampen and even offset the decline in Southern demand for Western varieties, A_{sw}, thus muting the competition effect. To examine this possibility, we have explored numerical examples given that the model is less tractable to solve analytically when wages are endogenous. As a baseline, when adopting parameter values similar to those presented in the proof of Proposition 1 and in the previous extension on Southern exporting, we continue to find that an improvement in host-country financial development reduces the share of sales to the local market, while raising the return and export-platform sales shares. For the baseline set of parameters that we work with, we moreover find that the competition effect is relatively insensitive to forces that might affect the magnitude of the endogenous wage response in South (specifically, the size of the labor force in West/East relative to that in South). We elaborate on these computational examples in Section C.3.

B.4 Multiple host countries

In a last extension, we show how the key insights can also be applied in a setting with multiple host countries. Consider a setup that maintains the structure of West and East from Section 2.1, but that now allows for two Southern countries ($s1$ and $s2$) as potential FDI hosts. Assume that country $s1$ is more financially developed than $s2$ ($0 < \eta_{s2} < \eta_{s1} < 1$), but that $s1$ and $s2$ are identical in all other respects. As in the baseline model, let $s1$ and $s2$ each have a differentiated-varieties industry whose products are in demand only in their respective domestic markets. We consider situations in which multinationals from West (likewise East) choose to undertake FDI in either $s1$ or $s2$, and subsequently use the Southern production plant to serve all four economies. Horizontal and return sales in either $s1$ or $s2$ are defined once again as sales in the local market and to the parent country (West) respectively; however, platform sales now comprise the sum of exports to East and to the other Southern country.

In Section C.4, we show that the competition effect – in particular, its implications for the horizontal, return and platform sales shares – directly applies to the variation across the host countries. Because of its higher financial development, $s1$ will feature more local firms than $s2$, and be a more competitive market environment for multinational affiliates based there, *ceteris paribus.* As a result, the horizontal sales share in $s1$ will be smaller than that in $s2$, while the return and platform sales shares will instead be larger. We further show how a comparison of affiliate sales levels between $s1$ and $s2$ can be made, once some additional structure is introduced that allows firms with the same productivity level to potentially undertake FDI in either host economy. This is the case when each prospective multinational observes an idiosyncratic profit shock in each host country that influences the location it ultimately chooses for its affiliate. In this setting, the qualitative predictions of Propositions 1 and 2 regarding sales levels extend to the cross-section of countries with different levels of financial development.

C Detailed Derivations

C.1 Model Extension: Home-bias in consumption

We establish that Proposition 1 in our baseline model continues to apply in this extension. We also provide the proof of Proposition 3, which allows for a differential response of platform versus return sales to changes in host-country financial development.

Model Setup. Recall that we modify the utility functions for $n = w, e$ (West and East) and for s (South) to:

$$U_n = (y_n)^{\mu^0} \prod_{k=1}^{K} \left(\sum_{j \in \{e,w\}} \left(\int_{\Omega_{nj}^k} x_{nj}^k(a)^{\alpha^k} \, dG_j^k(a) \right)^{\frac{\beta^k}{\alpha^k}} \right)^{\frac{\mu^k}{\beta^k}}, \text{ and}$$

$$U_s = (y_s)^{\mu^0} \prod_{k=1}^{K} \left(\sum_{j \in \{e,w,s\}} \left(\int_{\Omega_{sj}^k} x_{sj}^k(a)^{\alpha^k} \, dG_j^k(a) \right)^{\frac{\beta^k}{\alpha^k}} \right)^{\frac{\mu^k}{\beta^k}}.$$

where $0 < \beta^k < \alpha^k < 1$. We denote the elasticity of substitution for varieties from the same country by $\varepsilon^k = \frac{1}{1-\alpha^k}$, and the elasticity of substitution for varieties from different countries by $\theta^k = \frac{1}{1-\beta^k}$. Note that $\varepsilon^k > \theta^k > 1$, so that varieties from the same country are closer substitutes than varieties drawn from different countries.

As in the main paper, the exposition below focuses on a particular differentiated-varieties sector, so we drop the superscript k. Maximizing (C.1) and (C.1) subject to the standard budget constraints, one obtains that demand in country i for a variety from country j is: $x_{ij}(a) = A_{ij}p_{ij}(a)^{-\varepsilon}$. The aggregate market demand levels are now given by:

$$A_{ww} = A_{ee} = \frac{\mu E_n P_{ww}^{\varepsilon-\theta}}{P_{ww}^{1-\theta} + P_{ew}^{1-\theta}}, \tag{C.1}$$

$$A_{ew} = A_{we} = \frac{\mu E_n P_{ew}^{\varepsilon-\theta}}{P_{ww}^{1-\theta} + P_{ew}^{1-\theta}}, \tag{C.2}$$

$$A_{sw} = A_{se} = \frac{\mu E_s P_{sw}^{\varepsilon-\theta}}{P_{ss}^{1-\theta} + 2P_{sw}^{1-\theta}}, \quad \text{and} \tag{C.3}$$

$$A_{ss} = \frac{\mu E_s P_{ss}^{\varepsilon-\theta}}{P_{ss}^{1-\theta} + 2P_{sw}^{1-\theta}}. \tag{C.4}$$

In contrast to the baseline model, we no longer have $A_{ww} = A_{ew}$ and $A_{sw} = A_{ss}$. This is precisely due to the introduction of the additional elasticity of substitution, θ. In particular, when $\varepsilon = \theta$, the above collapses back to the demand expressions from our baseline model.

The rest of the equations for the equilibrium system remain the same as in the baseline model. For completeness, we reproduce them below:

$$a_D^{1-\varepsilon} = \frac{f_D}{(1-\alpha)A_{ww}(1/\alpha)^{1-\varepsilon}} \tag{C.5}$$

$$a_{XN}^{1-\varepsilon} = \frac{f_X}{(1-\alpha)A_{ew}(\tau/\alpha)^{1-\varepsilon}} \tag{C.6}$$

$$a_{XS}^{1-\varepsilon} = \frac{f_X}{(1-\alpha)A_{sw}(\tau/\alpha)^{1-\varepsilon}} \tag{C.7}$$

$$a_I^{1-\varepsilon} = \frac{f_I - f_D}{(1-\alpha)[A_{ww}((\frac{\tau\omega}{\alpha})^{1-\varepsilon} - (\frac{1}{\alpha})^{1-\varepsilon}) + A_{ew}((\frac{\tau\omega}{\alpha})^{1-\varepsilon} - (\frac{\tau}{\alpha})^{1-\varepsilon}) + A_{sw}((\frac{\omega}{\alpha})^{1-\varepsilon} - (\frac{\tau}{\alpha})^{1-\varepsilon})]} \tag{C.8}$$

$$a_S^{1-\varepsilon} = \frac{1}{\eta}\frac{R\phi f_S\omega}{(1-\alpha)A_{ss}(\omega/\alpha)^{1-\varepsilon}} \tag{C.9}$$

$$\begin{aligned} \delta f_{En} = {} & (1-\alpha)A_{ww}\left(\frac{1}{\alpha}\right)^{1-\varepsilon}(V_n(a_D) - V_n(a_I)) - f_D(G_n(a_D) - G_n(a_I)) \\ & +(1-\alpha)A_{ew}\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XN}) - V_n(a_I))f_X(G_n(a_{XN}) - G_n(a_I)) \\ & +(1-\alpha)A_{sw}\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XS}) - V_n(a_I)) - f_X(G_n(a_{XS}) - G_n(a_I)) \\ & +(1-\alpha)\left(A_{ww}\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} + A_{ew}\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} + A_{sw}\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}\right)V_n(a_I) - (f_I + 2f_X)G_n(a_I) \end{aligned} \tag{C.10}$$

$$\delta f_{Es}\omega = (1-\alpha)A_{ss}\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}V_s(a_S) - (R\phi + (1-\phi))f_S\omega G_s(a_S) \tag{C.11}$$

$$P_{ww}^{1-\varepsilon} = N_n\left[\left(\frac{1}{\alpha}\right)^{1-\varepsilon}(V_n(a_D) - V_n(a_I)) + \left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon}V_n(a_I)\right] \tag{C.12}$$

$$P_{ew}^{1-\varepsilon} = N_n\left[\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XN}) - V_n(a_I)) + \left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon}V_n(a_I)\right] \tag{C.13}$$

$$P_{sw}^{1-\varepsilon} = N_n\left[\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}(V_n(a_{XS}) - V_n(a_I)) + \left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}V_n(a_I)\right] \tag{C.14}$$

$$P_{ss}^{1-\varepsilon} = N_s\left[\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}V_s(a_S)\right] \tag{C.15}$$

The equilibrium is thus pinned down by the 15 equations (C.1)-(C.15) in the 15 endogeneous variables: A_{ww}, A_{ew}, A_{sw}, A_{ss}, a_D, a_{XN}, a_{XS}, a_I, a_S, N_n, N_s, P_{ww}, P_{ew}, P_{sw} and P_{ss}.

Proposition 1 continues to hold. It is clear that (C.9) and (C.11) once again pin down the equilibrium for South's differentiated-varieties industry. Since these equations are unchanged from the baseline model, this means that Lemma 1 holds, namely that $\frac{da_S}{d\eta} > 0$ and $\frac{dA_{ss}}{d\eta} < 0$.

We next show that a modified version of Lemma 2 now describes the subsequent impact on the industry equilibrium in West (and East):

Lemma 2A: *In the extended model with home-bias in consumption, (i)* $\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} < \frac{1}{a_I}\frac{da_I}{d\eta} < 0$*; (ii)* $\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta} > \frac{1}{a_D}\frac{da_D}{d\eta} > 0$*; (iii)* $\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} < 0$*; and (iv)* $\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} > \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} > 0$.

In response to a small increase in η, we now have the proportional shift in the a_{XN} cut-off exceeding that in the a_D cut-off, and hence the proportional increase in A_{ew} exceeding that in A_{ww}.

We proceed to prove this modified lemma. To provide a heuristic roadmap, we will take the remaining 13 equations that define the Western industry equilibrium – (C.1)-(C.3), (C.5)-(C.8), (C.10), and (C.12)-(C.15) –

and log-differentiate them. We then reduce the resulting system to a set of four equations in the four unknowns, $\frac{da_D}{a_D}$, $\frac{da_{XN}}{a_{XN}}$, $\frac{da_{XS}}{a_{XS}}$ and $\frac{da_I}{a_I}$. From this, we can determine the comparative statics with respect to η for the Western industry cut-offs, and hence for the other endogenous variables as well.

First, log-differentiating (C.5), (C.6) and (C.7) yields:

$$(\varepsilon-1)\frac{da_D}{a_D} = \frac{dA_{ww}}{A_{ww}}, \tag{C.16}$$

$$(\varepsilon-1)\frac{da_{XN}}{a_{XN}} = \frac{dA_{ew}}{A_{ew}}, \quad \text{and} \tag{C.17}$$

$$(\varepsilon-1)\frac{da_{XS}}{a_{XS}} = \frac{dA_{sw}}{A_{sw}}. \tag{C.18}$$

Since $\varepsilon > 1$, this implies: $sign\{\frac{da_D}{d\eta}\} = sign\{\frac{dA_{ww}}{d\eta}\}$, $sign\{\frac{da_{XN}}{d\eta}\} = sign\{\frac{dA_{ew}}{d\eta}\}$, and $sign\{\frac{da_{XS}}{d\eta}\} = sign\{\frac{dA_{sw}}{d\eta}\}$.

Similarly, log-differentiating (C.8) yields:

$$(\varepsilon-1)\frac{da_I}{a_I} = \frac{A_{ww}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right)\frac{dA_{ww}}{A_{ww}} + A_{ew}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)\frac{dA_{ew}}{A_{ew}} + A_{sw}\left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)\frac{dA_{sw}}{A_{sw}}}{A_{ww}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon}\right) + A_{ew}\left(\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right) + A_{sw}\left(\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon}\right)}.$$

We replace $\frac{dA_{ww}}{A_{ww}}$, $\frac{dA_{ew}}{A_{ew}}$ and $\frac{dA_{sw}}{A_{sw}}$ by the expressions in (C.16)-(C.18). Making use also of the expressions for A_{ww}, A_{ew} and A_{sw} from (C.1)-(C.3), and for $P_{ww}^{1-\varepsilon}$, $P_{ew}^{1-\varepsilon}$ and $P_{sw}^{1-\varepsilon}$ from (C.12)-(C.14), and simplifying extensively, one can show that:

$$\frac{da_I}{a_I} = \frac{\rho_1(1-\Delta_1)\frac{da_D}{a_D} + (1-\rho_1)(1-\Delta_2)\frac{da_{XN}}{a_{XN}} + \frac{1-\rho_2}{2}\frac{E_s}{E_n}(1-\Delta_3)\frac{da_{XS}}{a_{XS}}}{\rho_1(1-\Delta_1) + (1-\rho_1)(1-\Delta_2) + \frac{1-\rho_2}{2}\frac{E_s}{E_n}(1-\Delta_3)}, \tag{C.19}$$

where we now define: $\rho_1 = \frac{P_{ww}^{1-\theta}}{P_{ww}^{1-\theta}+P_{ew}^{1-\theta}}$ and $\rho_2 = \frac{P_{ss}^{1-\theta}}{P_{ss}^{1-\theta}+2P_{sw}^{1-\theta}}$. Note that in contrast to the proof for the baseline model, the definitions of ρ_1 and ρ_2 now involve θ, instead of ε. We nevertheless still have $\rho_1, \rho_2 \in (0,1)$. Recall also the following definitions, which we retain from the proof for the baseline model:

$$\Delta_1 = \frac{\left(\frac{1}{\alpha}\right)^{1-\varepsilon} V_n(a_D)}{\left(\frac{1}{\alpha}\right)^{1-\varepsilon} V_n(a_D) + (\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{1}{\alpha}\right)^{1-\varepsilon})V_n(a_I)}, \tag{C.20}$$

$$\Delta_2 = \frac{\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} V_n(a_{XN})}{\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} V_n(a_{XN}) + (\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon})V_n(a_I)}, \quad \text{and} \tag{C.21}$$

$$\Delta_3 = \frac{\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} V_n(a_{XS})}{\left(\frac{\tau}{\alpha}\right)^{1-\varepsilon} V_n(a_{XS}) + (\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} - \left(\frac{\tau}{\alpha}\right)^{1-\varepsilon})V_n(a_I)}. \tag{C.22}$$

Note that in the proof of Lemma 2, we showed that $1 > \Delta_1 > \Delta_2 > \Delta_3 > 0$.

Next, we differentiate the free-entry condition for West, (C.10). Following the algebraic manipulations used in the proof of Lemma 2, we once again obtain:

$$\rho_1\frac{da_D}{a_D} + (1-\rho_1)\frac{da_{XN}}{a_{XN}} + \frac{1-\rho_2}{2}\frac{E_s}{E_n}\frac{da_{XS}}{a_{XS}} = 0. \tag{C.23}$$

A quick implication is that the three cut-offs a_D, a_{XN} and a_{XS} cannot all move in the same direction.

We move on to log-differentiate the market demand expressions in (C.1)-(C.4):

$$\frac{dA_{ww}}{A_{ww}} = \left((1-\rho_1)\frac{\theta-1}{\varepsilon-1} - 1\right)\frac{dP_{ww}^{1-\varepsilon}}{P_{ww}^{1-\varepsilon}} - (1-\rho_1)\frac{\theta-1}{\varepsilon-1}\frac{dP_{ew}^{1-\varepsilon}}{P_{ew}^{1-\varepsilon}}, \tag{C.24}$$

$$\frac{dA_{ew}}{A_{ew}} = \left(\rho_1\frac{\theta-1}{\varepsilon-1} - 1\right)\frac{dP_{ew}^{1-\varepsilon}}{P_{ew}^{1-\varepsilon}} - \rho_1\frac{\theta-1}{\varepsilon-1}\frac{dP_{ww}^{1-\varepsilon}}{P_{ww}^{1-\varepsilon}}, \tag{C.25}$$

$$\frac{dA_{sw}}{A_{sw}} = \left(\rho_2\frac{\theta-1}{\varepsilon-1} - 1\right)\frac{dP_{sw}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}} - \rho_2\frac{\theta-1}{\varepsilon-1}\frac{dP_{ss}^{1-\varepsilon}}{P_{ss}^{1-\varepsilon}}, \quad \text{and} \tag{C.26}$$

$$\frac{dA_{ss}}{A_{ss}} = \left((1-\rho_2)\frac{\theta-1}{\varepsilon-1} - 1\right)\frac{dP_{ss}^{1-\varepsilon}}{P_{ss}^{1-\varepsilon}} - (1-\rho_2)\frac{\theta-1}{\varepsilon-1}\frac{dP_{sw}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}}. \tag{C.27}$$

Meanwhile, log-differentiating the ideal price indices (C.12)-(C.14) gives us:

$$\frac{dP_{ww}^{1-\varepsilon}}{P_{ww}^{1-\varepsilon}} = \frac{dN_n}{N_n} + (\kappa - \varepsilon + 1)\left(\Delta_1 \frac{da_D}{a_D} + (1-\Delta_1)\frac{da_I}{a_I}\right), \tag{C.28}$$

$$\frac{dP_{ew}^{1-\varepsilon}}{P_{ew}^{1-\varepsilon}} = \frac{dN_n}{N_n} + (\kappa - \varepsilon + 1)\left(\Delta_2 \frac{da_{XN}}{a_{XN}} + (1-\Delta_2)\frac{da_I}{a_I}\right), \quad \text{and} \tag{C.29}$$

$$\frac{dP_{sw}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}} = \frac{dN_n}{N_n} + (\kappa - \varepsilon + 1)\left(\Delta_3 \frac{da_{XS}}{a_{XS}} + (1-\Delta_3)\frac{da_I}{a_I}\right), \tag{C.30}$$

where we have made use of the fact that $\frac{aV_n'(a)}{V_n(a)} = \kappa - \varepsilon + 1$ for the Pareto distribution.

Using Cramer's Rule, we now invert (C.26) and (C.27) to obtain:

$$\frac{dP_{sw}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}} = \left(-\rho_2 \frac{\theta-1}{\varepsilon-\theta} - 1\right)\frac{dA_{sw}}{A_{sw}} + \rho_2 \frac{\theta-1}{\varepsilon-\theta}\frac{dA_{ss}}{A_{ss}}, \quad \text{and} \tag{C.31}$$

$$\frac{dP_{ss}^{1-\varepsilon}}{P_{ss}^{1-\varepsilon}} = \left(-(1-\rho_2)\frac{\theta-1}{\varepsilon-\theta} - 1\right)\frac{dA_{ss}}{A_{ss}} + (1-\rho_2)\frac{\theta-1}{\varepsilon-\theta}\frac{dA_{sw}}{A_{sw}}. \tag{C.32}$$

Setting (C.30) equal to (C.31) then implies:

$$\frac{dN_n}{N_n} = \rho_2 \frac{\theta-1}{\varepsilon-\theta}\frac{dA_{ss}}{A_{ss}} - \left[(\varepsilon-1)\left(\rho_2\frac{\theta-1}{\varepsilon-\theta}+1\right) + (\kappa-\varepsilon+1)\Delta_3\right]\frac{da_{XS}}{a_{XS}} - (\kappa-\varepsilon+1)(1-\Delta_3)\frac{da_I}{a_I}. \tag{C.33}$$

We now plug this expression for $\frac{dN_n}{N_n}$ into (C.28) and (C.29), and substitute the subsequent expressions for $\frac{dP_{ww}^{1-\varepsilon}}{P_{ww}^{1-\varepsilon}}$ and $\frac{dP_{ew}^{1-\varepsilon}}{P_{ew}^{1-\varepsilon}}$ into (C.24) and (C.25). Finally, replacing $\frac{dA_{ww}}{A_{ww}}$ and $\frac{dA_{ew}}{A_{ew}}$ with the expressions in terms of $\frac{da_D}{a_D}$ and $\frac{da_{XN}}{a_{XN}}$ from (C.16) and (C.17) respectively, one obtains after some rearrangement:

$$\begin{aligned}\frac{\rho_2}{\kappa-\varepsilon+1}\frac{\theta-1}{\varepsilon-\theta}\frac{dA_{ss}}{A_{ss}} &= \left[\left((1-\rho_1)\frac{\theta-1}{\varepsilon-1}-1\right)\Delta_1 - \frac{\varepsilon-1}{\kappa-\varepsilon+1}\right]\frac{da_D}{a_D} - (1-\rho_1)\frac{\theta-1}{\varepsilon-1}\Delta_2\frac{da_{XN}}{a_{XN}} \\ &\quad + \left[\frac{\varepsilon-1}{\kappa-\varepsilon+1}\left(\rho_2\frac{\theta-1}{\varepsilon-\theta}+1\right)+\Delta_3\right]\frac{da_{XS}}{a_{XS}} \\ &\quad + \left[(\Delta_1-\Delta_3) - (\Delta_1-\Delta_2)(1-\rho_1)\frac{\theta-1}{\varepsilon-1}\right]\frac{da_I}{a_I}, \quad \text{and}\end{aligned} \tag{C.34}$$

$$\begin{aligned}\frac{\rho_2}{\kappa-\varepsilon+1}\frac{\theta-1}{\varepsilon-\theta}\frac{dA_{ss}}{A_{ss}} &= -\rho_1\frac{\theta-1}{\varepsilon-1}\Delta_1\frac{da_D}{a_D} + \left[\left(\rho_1\frac{\theta-1}{\varepsilon-1}-1\right)\Delta_2 - \frac{\varepsilon-1}{\kappa-\varepsilon+1}\right]\frac{da_{XN}}{a_{XN}} \\ &\quad + \left[\frac{\varepsilon-1}{\kappa-\varepsilon+1}\left(\rho_2\frac{\theta-1}{\varepsilon-\theta}+1\right)+\Delta_3\right]\frac{da_{XS}}{a_{XS}} \\ &\quad + \left[(\Delta_2-\Delta_3) + (\Delta_1-\Delta_2)\rho_1\frac{\theta-1}{\varepsilon-1}\right]\frac{da_I}{a_I}.\end{aligned} \tag{C.35}$$

(C.19), (C.23), (C.34), and (C.35) give us four equations in the four unknowns, $\frac{da_D}{a_D}$, $\frac{da_{XN}}{a_{XN}}$, $\frac{da_{XS}}{a_{XS}}$ and $\frac{da_I}{a_I}$. To pin down the comparative statics explicitly, note that equating (C.35) and (C.34) implies:

$$\frac{da_I}{a_I} = \frac{1}{\Delta_1-\Delta_2}\left[\left(\Delta_1 + \frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)\frac{da_D}{d_D} - \left(\Delta_2 + \frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)\frac{da_{XN}}{d_{XN}}\right]. \tag{C.36}$$

Meanwhile, using (C.23) to eliminate $\frac{da_{XS}}{a_{XS}}$ from (C.19) delivers:

$$\frac{da_I}{a_I} = -\frac{\rho_1(\Delta_1-\Delta_3)\frac{da_D}{a_D} + (1-\rho_1)(\Delta_2-\Delta_3)\frac{da_{XN}}{a_{XN}}}{\rho_1(1-\Delta_1) + (1-\rho_1)(1-\Delta_2) + \frac{1-\rho_2}{2}\frac{E_s}{E_n}(1-\Delta_3)}. \tag{C.37}$$

For convenience, let us define: $\Delta_d = \rho_1(1-\Delta_1) + (1-\rho_1)(1-\Delta_2) + \frac{1-\rho_2}{2}\frac{E_s}{E_n}(1-\Delta_3) > 0$, which is the denominator in (C.37). Then, setting (C.36) equal to (C.37) and rearranging, one obtains:

$$\begin{aligned}0 &= \left[\rho_1(\Delta_1-\Delta_3)(\Delta_1-\Delta_2) + \Delta_d\left(\Delta_1 + \frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)\right]\frac{da_D}{a_D} \\ &\quad + \left[(1-\rho_1)(\Delta_2-\Delta_3)(\Delta_1-\Delta_2) - \Delta_d\left(\Delta_2 + \frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)\right]\frac{da_{XN}}{a_{XN}}.\end{aligned} \tag{C.38}$$

Since $\Delta_1 - \Delta_2, \Delta_1 - \Delta_3 > 0$, it follows that the coefficient of $\frac{da_D}{a_D}$ in (C.38) is positive. Moreover, using the definition of Δ_d, one can see that the coefficient of $\frac{da_{XN}}{a_{XN}}$ is strictly smaller than: $(1-\rho_1)(\Delta_2-\Delta_3)(\Delta_1-\Delta_2) - (1-\rho_1)(1-\Delta_2)\Delta_2$, which itself is already negative, since: $1-\Delta_2 > \Delta_1 - \Delta_2 > 0$, and $\Delta_2 > \Delta_2 - \Delta_3 > 0$. Thus, the coefficient of $\frac{da_{XN}}{a_{XN}}$ in (C.38) is negative. Since the linear combination in (C.38) is equal to 0, it follows that $sign\{\frac{da_D}{d\eta}\} = sign\{\frac{da_{XN}}{d\eta}\}$.

We require one more equation in $\frac{da_D}{a_D}$ and $\frac{da_{XN}}{a_{XN}}$ in order to pin down their common sign. For this, substitute the expression for $\frac{da_I}{a_I}$ from (C.37) and that for $\frac{da_{XS}}{a_{XS}}$ from (C.23) into (C.34) to obtain:

$$\begin{aligned}\frac{\rho_2}{\kappa-\varepsilon+1}\frac{\theta-1}{\varepsilon-\theta}\frac{dA_{ss}}{A_{ss}} \quad = \quad & \Bigg[\left(\left((1-\rho_1)\frac{\theta-1}{\varepsilon-1}-1\right)\Delta_1 - \frac{2\rho_1}{1-\rho_2}\frac{E_n}{E_s}\left(\frac{\varepsilon-1}{\kappa-\varepsilon+1}\left(\rho_2\frac{\theta-1}{\varepsilon-\theta}+1\right)+\Delta_3\right)\right. \\ & \left. -\frac{\varepsilon-1}{\kappa-\varepsilon+1} - \left((\Delta_1-\Delta_3)-(\Delta_1-\Delta_2)(1-\rho_1)\frac{\theta-1}{\varepsilon-1}\right)\frac{\rho_1(\Delta_1-\Delta_3)}{\Delta_d}\right]\frac{da_D}{a_D} \\ & +\Bigg[-(1-\rho_1)\frac{\theta-1}{\varepsilon-1}\Delta_2 - \frac{2(1-\rho_1)}{1-\rho_2}\frac{E_n}{E_s}\left(\frac{\varepsilon-1}{\kappa-\varepsilon+1}\left(\rho_2\frac{\theta-1}{\varepsilon-\theta}+1\right)+\Delta_3\right) \\ & -\left((\Delta_1-\Delta_3)-(\Delta_1-\Delta_2)(1-\rho_1)\frac{\theta-1}{\varepsilon-1}\right)\frac{(1-\rho_1)(\Delta_2-\Delta_3)}{\Delta_d}\Bigg]\frac{da_{XN}}{a_{XN}}. \end{aligned} \tag{C.39}$$

Note that $(\Delta_1-\Delta_3)-(\Delta_1-\Delta_2)(1-\rho_1)\frac{\theta-1}{\varepsilon-1} > 0$, since: $\Delta_1 - \Delta_3 > \Delta_1 - \Delta_2 > 0$, $1-\rho_1 \in (0,1)$, and $\frac{\theta-1}{\varepsilon-1} \in (0,1)$. These conditions also imply that: $(1-\rho_1)\frac{\theta-1}{\varepsilon-1} - 1 < 0$. It is then straightforward to see that the coefficients of both $\frac{da_D}{a_D}$ and $\frac{da_{XN}}{a_{XN}}$ in (C.39) are negative. From Lemma 1, recall that $\frac{dA_{ss}}{d\eta} < 0$. It follows then from (C.39) that $sign\{\frac{da_D}{d\eta}\} = sign\{\frac{da_{XN}}{d\eta}\} > 0$.

Rearranging (C.38) now implies:

$$\frac{\frac{1}{a_D}\frac{da_D}{d\eta}}{\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}} = \frac{-(1-\rho_1)(\Delta_2-\Delta_3)(\Delta_1-\Delta_2)+\Delta_d\left(\Delta_2+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)}{\rho_1(\Delta_1-\Delta_3)(\Delta_1-\Delta_2)+\Delta_d\left(\Delta_1+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)}. \tag{C.40}$$

It is easy to verify that the numerator of (C.40) is positive but smaller than the denominator; in particular, this is a consequence of $\Delta_1 > \Delta_2$. It follows that $\frac{1}{a_D}\frac{da_D}{d\eta} / \frac{1}{a_{XN}}\frac{da_{XN}}{d\eta} \in (0,1)$, so that: $\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta} > \frac{1}{a_D}\frac{da_D}{d\eta} > 0$, as stated in part (i) of Lemma 2A. Part (iii) of the lemma then holds immediately from (C.16) and (C.17).

As for part (ii) of the lemma, observe that (C.23) implies:

$$\frac{da_{XS}}{a_{XS}} = -\frac{2}{1-\rho_2}\frac{E_n}{E_s}\left(\rho_1\frac{da_D}{a_D}+(1-\rho_1)\frac{da_{XN}}{a_{XN}}\right) < 0. \tag{C.41}$$

At the same time, it is clear from (C.37) that $\frac{da_I}{a_I} < 0$. Now, subtracting (C.41) from (C.37) yields:

$$\frac{da_I}{a_I} - \frac{da_{XS}}{a_{XS}} = \left(-\frac{\Delta_1-\Delta_3}{\Delta_d}+\frac{2}{1-\rho_2}\frac{E_n}{E_s}\right)\rho_1\frac{da_D}{a_D} + \left(-\frac{\Delta_2-\Delta_3}{\Delta_d}+\frac{2}{1-\rho_2}\frac{E_n}{E_s}\right)(1-\rho_1)\frac{da_{XN}}{a_{XN}}. \tag{C.42}$$

One can check directly that: $\frac{2}{1-\rho_2}\frac{E_n}{E_s}\Delta_d > 1-\Delta_3 > \Delta_1-\Delta_3, \Delta_2-\Delta_3$. The coefficients of $\frac{da_D}{a_D}$ and $\frac{da_{XN}}{a_{XN}}$ from this last equation are thus both positive, from which we can conclude that: $\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} < \frac{1}{a_I}\frac{da_I}{d\eta} < 0$. Finally, part (iv) follows from the fact that $\frac{da_{XS}}{a_{XS}}$ and $\frac{dA_{sw}}{A_{sw}}$ share the same sign (from (C.18)). This concludes the proof of Lemma 2A.

We now proceed to establish that Proposition 1 continues to apply in the extended model with home-bias in the utility specification. Recall the definitions of $HOR(a)$, $PLA(a)$ and $RET(a)$ from Section 2.2. From these, it is clear that the effects of η on the affiliate-level sales values are pinned down respectively by the derivatives of A_{sw}, A_{ew} and A_{ww} with respect to η. Lemma 2A then implies that when Southern financial development improves, $HOR(a)$ falls (since $\frac{dA_{sw}}{d\eta} < 0$), $PLA(a)$ increases (since $\frac{dA_{ew}}{d\eta} > 0$), and $RET(a)$ increases (since $\frac{dA_{ww}}{d\eta} > 0$). This establishes part (i) of the proposition.

Next, recall the expressions for the sales shares by destination listed in equations (2.23)-(2.25). One can see that $\frac{d}{d\eta}\frac{HORI(a)}{TOT(a)} < 0$, since both $\frac{A_{ww}}{A_{sw}}$ and $\frac{A_{ew}}{A_{sw}}$ increase with η. On the other hand, we have $\frac{d}{d\eta}\frac{PLAT(a)}{TOT(a)} > 0$, since both $\frac{A_{sw}}{A_{ew}}$ and $\frac{A_{ww}}{A_{ew}}$ are decreasing in η. (That $\frac{d}{d\eta}\frac{A_{ww}}{A_{ew}} < 0$ follows from $\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} > \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} > 0$.) It remains to show that $\frac{d}{d\eta}\frac{RET(a)}{TOT(a)} > 0$ as well. From equation (2.25), it suffices to show that $\tau^{\varepsilon-1}\frac{A_{sw}}{A_{ww}} + \frac{A_{ew}}{A_{ww}}$ decreases with η:

$$\begin{aligned}
\frac{d}{d\eta}\left(\tau^{\varepsilon-1}\frac{A_{sw}}{A_{ww}} + \frac{A_{ew}}{A_{ww}}\right) &\propto \tau^{\varepsilon-1}A_{sw}\left(\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} - \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}\right) + A_{ew}\left(\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} - \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}\right) \\
&\propto \tau^{\varepsilon-1}\frac{A_{sw}}{A_{ew}}\left(\frac{1}{a_{XS}}\frac{da_{XS}}{d\eta} - \frac{1}{a_D}\frac{da_D}{d\eta}\right) + \left(\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta} - \frac{1}{a_D}\frac{da_D}{d\eta}\right),
\end{aligned}$$

where '$\propto$' denotes equality up to a positive multiplicative term. Note that we have used (C.16)-(C.18) in the last step above. We now replace $\frac{da_{XS}}{d\eta}$ using the expression in (C.41). Also, based on the definitions from (C.1) and (C.2), one can show that: $\frac{A_{sw}}{A_{ew}} = \frac{E_s}{E_n}\frac{1-\rho_2}{2(1-\rho_1)}\frac{P_{ew}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}}$. Performing these substitutions and rearranging, we obtain:

$$\frac{d}{d\eta}\left(\tau^{\varepsilon-1}\frac{A_{sw}}{A_{ww}} + \frac{A_{ew}}{A_{ww}}\right) \propto -\left[1 + \tau^{\varepsilon-1}\frac{A_{sw}}{A_{ew}}\left(\frac{E_n}{E_s}\frac{2\rho_1}{1-\rho_2} + 1\right)\right]\frac{1}{a_D}\frac{da_D}{d\eta} + \left[1 - \tau^{\varepsilon-1}\frac{P_{ew}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}}\right]\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}.$$

In this last equation, the coefficient of $\frac{1}{a_D}\frac{da_D}{d\eta}$ is clearly negative. As for the coefficient of $\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}$, using the expressions for $P_{ew}^{1-\varepsilon}$ and $P_{sw}^{1-\varepsilon}$ from (C.13) and (C.14), we have:

$$\begin{aligned}
1 - \tau^{\varepsilon-1}\frac{P_{ew}^{1-\varepsilon}}{P_{sw}^{1-\varepsilon}} &= 1 - \tau^{\varepsilon-1}\left[\frac{\tau^{1-\varepsilon}V_N(a_{XN}) + ((\tau\omega)^{1-\varepsilon} - \tau^{1-\varepsilon})V_N(a_I)}{\tau^{1-\varepsilon}V_N(a_{XS}) + (\omega^{1-\varepsilon} - \tau^{1-\varepsilon})V_N(a_I)}\right] \\
&= \frac{\tau^{1-\varepsilon}(V_N(a_{XS}) - V_N(a_I)) - (V_N(a_{XN}) - V_N(a_I))}{\tau^{1-\varepsilon}V_N(a_{XS}) + (\omega^{1-\varepsilon} - \tau^{1-\varepsilon})V_N(a_I)} \\
&< \frac{(\tau^{1-\varepsilon} - 1)(V_N(a_{XN}) - V_N(a_I))}{\tau^{1-\varepsilon}V_N(a_{XS}) + (\omega^{1-\varepsilon} - \tau^{1-\varepsilon})V_N(a_I)} \\
&< 0.
\end{aligned}$$

The second-to-last step relies on the fact that $V_N(a_{XN}) > V_N(a_{XS})$ (since $a_{XN} > a_{XS}$), while the last step follows from $\tau^{1-\varepsilon} < 1$ and $V_N(a_{XN}) > V_N(a_I)$ (since $a_{XN} > a_I$). The coefficient of $\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}$ is thus negative as well. Since $\frac{da_D}{d\eta}, \frac{da_{XN}}{d\eta} > 0$, this implies: $\frac{d}{d\eta}\left(\tau^{\varepsilon-1}\frac{A_{sw}}{A_{ww}} + \frac{A_{ew}}{A_{ww}}\right) < 0$. Hence, $\frac{RET(a)}{TOT(a)}$ increases with η.

It remains for us to prove part (iii) of the proposition, which contains the implications of host-country financial development for the various aggregate measures of multinational activity. To pin down the effect on N_n, we solve for $\frac{dN_n}{N_n}$ from (C.29). First, applying Cramer's Rule to (C.24) and (C.25), we have:

$$\frac{dP_{ew}^{1-\varepsilon}}{P_{ew}^{1-\varepsilon}} = \rho_1\frac{\theta-1}{\varepsilon-\theta}\left(\frac{dA_{ww}}{A_{ww}} - \frac{dA_{ew}}{A_{ew}}\right) - \frac{dA_{ew}}{A_{ew}} = (\varepsilon-1)\left[\rho_1\frac{\theta-1}{\varepsilon-\theta}\left(\frac{da_D}{a_D} - \frac{da_{XN}}{a_{XN}}\right) - \frac{da_{XN}}{a_{XN}}\right]. \quad \text{(C.43)}$$

Substituting from (C.43) into (C.29), replacing $\frac{da_I}{a_I}$ with the expression from (C.36), and rearranging yields:

$$\begin{aligned}
\frac{1}{\kappa-\varepsilon+1}\frac{dN_n}{N_n} &= \left[\rho_1\frac{\theta-1}{\varepsilon-\theta}\frac{\varepsilon-1}{\kappa-\varepsilon+1} - \frac{1-\Delta_2}{\Delta_1-\Delta_2}\left(\Delta_1 + \frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)\right]\frac{da_D}{a_D} \\
&\quad + \left[-\left(\rho_1\frac{\theta-1}{\varepsilon-\theta} + 1\right)\frac{\varepsilon-1}{\kappa-\varepsilon+1} - \Delta_2 + \frac{1-\Delta_2}{\Delta_1-\Delta_2}\left(\Delta_2 + \frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)\right]\frac{da_{XN}}{a_{XN}}.
\end{aligned}$$

To determine the sign of $\frac{dN_n}{N_n}$, divide the right-hand side of the above by $\frac{da_{XN}}{a_{XN}}$, and substitute in the expression for $\frac{da_D}{a_D}/\frac{da_{XN}}{d_{XN}}$ from (C.40). After some algebra, one can show that $sign\{\frac{dN_n}{d\eta}\}$ is given by the sign of:

$$
\begin{aligned}
& -\left(\Delta_2+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\right)\left[\Delta_d\left(\Delta_1+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)+\rho_1(\Delta_1-\Delta_2)(\Delta_1-\Delta_3)\right] \\
& \quad -\rho_1\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\theta-1}{\varepsilon-\theta}(\Delta_1-\Delta_2)\left[\rho_1(\Delta_1-\Delta_3)+(1-\rho_1)(\Delta_2-\Delta_3)+\Delta_d\right] \\
& \quad +(1-\Delta_2)\left[\left(\Delta_2+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)\rho_1(\Delta_1-\Delta_3)+\left(\Delta_1+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)(1-\rho_1)(\Delta_2-\Delta_3)\right] \\
< & -\left(\Delta_2+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\right)\left[(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\left(\Delta_1+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)+\rho_1(\Delta_1-\Delta_2)(\Delta_1-\Delta_3)\right] \\
& \quad -\rho_1\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\theta-1}{\varepsilon-\theta}(\Delta_1-\Delta_2)(1-\Delta_3) \\
& \quad +(1-\Delta_2)\left[\left(\Delta_2+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)\rho_1(\Delta_1-\Delta_3)+\left(\Delta_1+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)(1-\rho_1)(\Delta_2-\Delta_3)\right], \qquad \text{(C.44)}
\end{aligned}
$$

where the inequality comes from applying: $\Delta_d > \rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2)$. We now collect all the terms in (C.44) in which $\frac{\varepsilon-1}{\kappa-\varepsilon+1}$ does not appear. These are:

$$
\begin{aligned}
& -\Delta_2\left[(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\Delta_1+\rho_1(\Delta_1-\Delta_2)(\Delta_1-\Delta_3)\right]+(1-\Delta_2)\left[\Delta_2\rho_1(\Delta_1-\Delta_3)+\Delta_1(1-\rho_1)(\Delta_2-\Delta_3)\right] \\
= & -\Delta_3\left[\rho_1\Delta_2(1-\Delta_1)+(1-\rho_1)\Delta_1(1-\Delta_2)\right] \\
< & \ 0.
\end{aligned}
$$

This term is negative, since $\rho_1, \Delta_1, \Delta_2, \Delta_3 \in (0,1)$. Similarly, we collect the remaining terms in (C.44), all of which involve $\frac{\varepsilon-1}{\kappa-\varepsilon+1}$, as follows:

$$
\begin{aligned}
& -\frac{\varepsilon-1}{\kappa-\varepsilon+1}\left[(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\left(\Delta_1+\frac{\varepsilon-1}{\kappa-\varepsilon+1}\frac{\varepsilon-1}{\varepsilon-\theta}\right)+\rho_1(\Delta_1-\Delta_2)(\Delta_1-\Delta_3)\right. \\
& \quad +\Delta_2(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\frac{\varepsilon-1}{\varepsilon-\theta} \\
& \quad \left.+\rho_1\frac{\theta-1}{\varepsilon-\theta}(\Delta_1-\Delta_2)(1-\Delta_3)-\frac{\varepsilon-1}{\varepsilon-\theta}(1-\Delta_2)(\rho_1(\Delta_1-\Delta_3)+(1-\rho_1)(\Delta_2-\Delta_3))\right] \\
< & -\frac{\varepsilon-1}{\kappa-\varepsilon+1}\left[(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\Delta_1+\rho_1(\Delta_1-\Delta_2)(\Delta_1-\Delta_3)+\frac{\theta-1}{\varepsilon-\theta}\rho_1(\Delta_1-\Delta_2)(1-\Delta_3)\right. \\
& \quad \left.+\Delta_2(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\frac{\varepsilon-1}{\varepsilon-\theta}-\frac{\varepsilon-1}{\varepsilon-\theta}(1-\Delta_2)(\rho_1(\Delta_1-\Delta_3)+(1-\rho_1)(\Delta_2-\Delta_3))\right] \\
= & -\frac{\varepsilon-1}{\kappa-\varepsilon+1}\left[\rho_1(1-\Delta_1)\Delta_2+(1-\rho_1)\Delta_1(1-\Delta_2)+\frac{\varepsilon-1}{\varepsilon-\theta}\Delta_3(\rho_1(1-\Delta_1)+(1-\rho_1)(1-\Delta_2))\right] \\
< & \ 0,
\end{aligned}
$$

since $\frac{\varepsilon-1}{\kappa-\varepsilon+1} > 0$. This completes the proof that $\frac{dN_n}{d\eta} < 0$. As $\frac{da_I}{d\eta}$ is also negative, we thus have $\frac{1}{N_n}\frac{dN_n}{d\eta} + \kappa\frac{1}{a_I}\frac{da_I}{d\eta} < 0$, so that $\frac{d}{d\eta}\log N_n G_n(a_I) < 0$.

Finally, we derive the effects of changes in η on the aggregate sales variables defined in equations (2.20)-(2.22). Since $V_n(a)$ is an increasing function for all $a \in (0, \bar{a}_n)$, an improvement in η leads to a decrease in a_I, and hence in $V_n(a_I)$. Also, we have just seen that N_n decreases in η. To show that HOR, PLA and RET all decline in η, it therefore suffices to prove that PLA is declining in η, since $\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} > \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}, \frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta}$. From (2.21), we have:

$$
\begin{aligned}
\frac{d}{d\eta}\ln(PLA) &= \frac{1}{N_n}\frac{dN_n}{d\eta}+\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta}+\frac{V_N'(a_I)a_I}{V_N(a_I)}\frac{1}{a_I}\frac{da_I}{d\eta} \\
&= (\varepsilon-1)\left[\rho_1\frac{\theta-1}{\varepsilon-\theta}\left(\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}-\frac{1}{a_D}\frac{da_D}{d\eta}\right)-\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}\right] \\
&\quad -(\kappa-\varepsilon+1)\left(\Delta_2\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}+(1-\Delta_2)\frac{1}{a_I}\frac{da_I}{d\eta}\right)+(\varepsilon-1)\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}+(\kappa-\varepsilon+1)\frac{1}{a_I}\frac{da_I}{d\eta} \\
&= -(\varepsilon-1)\rho_1\frac{\theta-1}{\varepsilon-\theta}\left(\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}-\frac{1}{a_D}\frac{da_D}{d\eta}\right)-(\kappa-\varepsilon+1)\Delta_2\left(\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta}-\frac{1}{a_I}\frac{da_I}{d\eta}\right) \\
&< 0.
\end{aligned}
$$

To get from the first line above to the second, we have used the expression for $\frac{dN_n}{d\eta}$ from (C.29), and substituted for $\frac{dP_{ew}^{1-\varepsilon}}{d\eta}$ using (C.43). We have also used (C.16) and (C.17) to substitute for $\frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}$ and $\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta}$ wherever these terms appear. Finally, we have used the fact that $\frac{V_N'(a_I)a_I}{V_N(a_I)} = \kappa - \varepsilon + 1$ for the Pareto distribution. The last step establishing that $\frac{d}{d\eta}\ln(PLA) < 0$ follows from $\frac{1}{a_{XN}}\frac{da_{XN}}{d\eta} > \frac{1}{a_D}\frac{da_D}{d\eta} > \frac{1}{a_I}\frac{da_I}{d\eta}$, bearing in mind that $\theta - 1 > 0$ and $\kappa - \varepsilon + 1 > 0$. Thus, when η increases, the contraction in the extensive margin captured by the fall in N_n and $V_N(a_I)$ is larger in magnitude than the increase in sales on the intensive margin due to the rise in the demand level, A_{ew}. This concludes our proof that Proposition 1 continues to hold in the extended model with home-bias in consumption. ■

Proof of Proposition 3. For part (i) of the proposition, from the definitions of $PLA(a)$ and $RET(a)$, we have:

$$\frac{d}{d\eta}(PLA(a) - RET(a)) = (1-\alpha)\left(\frac{\tau a\omega}{\alpha}\right)^{1-\varepsilon} A_{ww}\left(\frac{A_{ew}}{A_{ww}}\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} - \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}\right).$$

We show first that $\frac{A_{ew}}{A_{ww}} > 1$. From (C.1) and (C.2), we have:

$$\frac{A_{ew}}{A_{ww}} = \left[\frac{V_N(a_D) + ((\tau\omega)^{1-\varepsilon} - 1)V_N(a_I)}{\tau^{1-\varepsilon}V_N(a_{XN}) + ((\tau\omega)^{1-\varepsilon} - \tau^{1-\varepsilon})V_N(a_I)}\right]^{\frac{\varepsilon-\theta}{\varepsilon-1}}. \tag{C.45}$$

Observe that:

$$\begin{aligned}
& V_N(a_D) + ((\tau\omega)^{1-\varepsilon} - 1)V_N(a_I) - \left(\tau^{1-\varepsilon}V_N(a_{XN}) + ((\tau\omega)^{1-\varepsilon} - \tau^{1-\varepsilon})V_N(a_I)\right) \\
= \;& V_N(a_D) - V_N(a_I) - \tau^{1-\varepsilon}(V_N(a_{XN}) - V_N(a_I)) \\
> \;& (1 - \tau^{1-\varepsilon})(V_N(a_{XN}) - V_N(a_I)) \\
> \;& 0,
\end{aligned}$$

where the second-to-last step uses the fact that $V_N(a_D) > V_N(a_{XN})$ (since $a_D > a_{XN}$), while the final step holds because $\tau^{1-\varepsilon} < 1$. Since the exponent, $\frac{\varepsilon-\theta}{\varepsilon-1}$, is positive (as $\varepsilon > \theta > 1$), it follows that $\frac{A_{ew}}{A_{ww}} > 1$, as claimed. We thus have:

$$\frac{d}{d\eta}(PLA(a) - RET(a)) \;>\; (1-\alpha)\left(\frac{\tau a\omega}{\alpha}\right)^{1-\varepsilon} A_{ww}\left(\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} - \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}\right) \;>\; 0,$$

since $\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} > \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}$ from Lemma 2A.

For part (ii) of the proposition, applying the quotient rule to the expressions for $\frac{PLA(a)}{TOT(a)}$ and $\frac{RET(a)}{TOT(a)}$ from (2.24) and (2.25) respectively, one obtains after some simplification that:

$$\begin{aligned}
\frac{d}{d\eta}\left[\frac{PLA(a)}{TOT(a)} - \frac{RET(a)}{TOT(a)}\right] \;\propto\; & \tau^{\varepsilon-1}\frac{A_{sw}}{A_{ew}}\left(1 - \frac{A_{ew}}{A_{ww}}\right)\frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta} + 2\left(\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} - \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}\right) \\
& + \tau^{\varepsilon-1}\frac{A_{sw}}{A_{ew}}\left(\frac{A_{ew}}{A_{ww}}\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} - \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta}\right) \\
> \;& 0,
\end{aligned}$$

where the last inequality follows from: $\frac{1}{A_{ew}}\frac{dA_{ew}}{d\eta} > \frac{1}{A_{ww}}\frac{dA_{ww}}{d\eta} > 0 > \frac{1}{A_{sw}}\frac{dA_{sw}}{d\eta}$ (Lemma 2A), and $\frac{A_{ew}}{A_{ww}} > 1$.

Finally, part (iii) of the proposition can be established using the definitions for PLA and RET in equations (2.21) and (2.22), and following analogous steps to the proof used for part (i) above. ■

C.2 Model Extension: Southern exports

Model Setup. To incorporate Southern exporting into our model, consumers in West and East need to have a positive demand level for Southern varieties. We therefore introduce Southern varieties (subscript s) into the utility function for $n = w, e$ (West and East) as follows:

$$U_n = (y_n)^{\mu^0} \prod_{k=1}^{K} \left(\sum_{j \in \{e,w,s\}} \int_{\Omega_{nj}^k} x_{nj}^k(a)^{\alpha^k} \; dG_j^k(a) \right)^{\frac{\mu^k}{\alpha^k}}, \tag{C.46}$$

The utility function for Southern consumers remains as in the baseline model, and is reproduced below:

$$U_s = (y_s)^{\mu^0} \prod_{k=1}^{K} \left(\sum_{j \in \{e,w,s\}} \int_{\Omega_{sj}^k} x_{sj}^k(a)^{\alpha^k} \; dG_j^k(a) \right)^{\frac{\mu^k}{\alpha^k}}, \tag{C.47}$$

Recall that $0 < \alpha^k, \mu^k < 1$, and that $\varepsilon^k = \frac{1}{1-\alpha^k} > 1$.

We focus the rest of the description of the setup on a given differentiated-varieties sector, dropping the superscript k to avoid cluttering the notation. Together with the standard budget constraints, (C.46) and (C.47) imply that the demand in country i for a country-j variety is given by: $x_{ij}(a) = A_{ij} p_{ij}(a)^{-\varepsilon}$, where the aggregate market demand levels are:

$$A_{ww} = A_{ee} = A_{ew} = A_{we} = A_{ws} = A_{es} = \frac{\mu L_n}{P_{ww}^{1-\varepsilon} + P_{we}^{1-\varepsilon} + P_{ws}^{1-\varepsilon}}, \quad \text{and} \tag{C.48}$$

$$A_{sw} = A_{se} = A_{ss} = \frac{\mu \omega L_s}{P_{ss}^{1-\varepsilon} + 2P_{sw}^{1-\varepsilon}}. \tag{C.49}$$

Note that we have introduced the notation A_{ws} and A_{es} to denote respectively the aggregate demand levels in West and East for Southern varieties. We analogously define $P_{ws} = P_{es}$ to be the ideal price index for the Southern varieties that are consumed in West and East respectively.

Turning to the structure of the differentiated-varieties industry in West/East, we retain here the setup from our baseline model. This means that the productivity cut-off expressions that were listed in equations (A.15)-(A.18) earlier continue to apply.

As for the differentiated-varieties industry in South, firms can enter as before into production for the domestic market by paying a fixed cost equal to f_S units of local labor. These firms face financial constraints and the corresponding no-default condition from Section 2.3 of the baseline model implies that the productivity cut-off for entering into production, $a_S^{1-\varepsilon}$, is given once again by (2.5). Southern firms now have the further option to export their output to West and East if they are sufficiently productive. We assume that this involves a familiar iceberg trade cost, $\tau > 1$, while also incurring a fixed cost of $f_{X,ws}$ units of Southern labor per market to commence exporting. This Southern exporting activity is however affected by credit constraints, as South is the less financially-developed country and prospective exporters need to raise the financing for a fraction ϕ of $f_{X,ws}$ from Southern financial markets. In the event of a default, we assume that Southern financiers are able to appropriate only a fraction $\eta \in (0, 1)$ of the operating profits from exporting (revenues less variable costs) from the firm. The corresponding no-default condition is thus:

$$\eta(1-\alpha)A_{ws}(\tau\omega/\alpha)^{1-\varepsilon} < R\phi f_{X,ws}\omega.$$

A simple rearrangement of the above implies the following cut-off, $a_{X,ws}^{1-\varepsilon}$, for exporting to commence:

$$a_{X,ws}^{1-\varepsilon} = \frac{1}{\eta} \frac{R\phi f_{X,ws}\omega}{(1-\alpha)A_{ws}(\tau\omega/\alpha)^{1-\varepsilon}}. \tag{C.50}$$

Note that we assume here that a Southern firm with productivity level $a_{X,ws}^{1-\varepsilon}$ would earn positive profits from exporting if it had access to credit, namely that: $(1-\alpha)A_{ws}(\tau\omega a_{X,ws}/\alpha)^{1-\varepsilon} - (R\phi + (1-\phi))f_{X,ws}\omega > 0$. There is thus a margin of Southern firms with productivity just lower than $a_{X,ws}^{1-\varepsilon}$ who would earn positive profits if given the opportunity to export, but are unable to do so due to the inherent default risk. In other words, credit constraints are binding in denying a margin of Southern firms that would be profitable at exporting to West/East the opportunity to do so. We adopt the natural ordering of productivity cut-offs, $0 < a_D^{1-\varepsilon} < a_{X,ws}^{1-\varepsilon}$, so that only the most productive Southern firms are able to engage in direct exporting. Given the symmetry between West and East, firms with $a^{1-\varepsilon} > a_{X,ws}^{1-\varepsilon}$ will export to both of these countries.

We close the model by spelling out the free entry conditions and the expressions for the ideal price indices. As the industry structure in West/East is unchanged, the free entry condition there continues to be given by (A.5). On the other hand, the corresponding condition for South now needs to take into further account the *ex ante* expected profits from exporting:

$$f_{Es}\omega = \frac{1}{\delta}\left[(1-\alpha)A_{ss}\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon}V_s(a_S) - (R\phi + (1-\phi))f_S\omega G_s(a_S) \right.$$
$$\left. \ldots + (1-\alpha)(A_{ws}+A_{es})\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon}V_s(a_{X,ws}) - 2(R\phi+(1-\phi))f_X\omega G_s(a_{X,ws})\right]. \quad \text{(C.51)}$$

For the ideal price indices, $P_{ww}^{1-\varepsilon}$, $P_{ew}^{1-\varepsilon}$, $P_{sw}^{1-\varepsilon}$ and $P_{ss}^{1-\varepsilon}$ continue to be given by (A.7)-(A.10). There is one additional index for the prices of Southern varieties that are exported in West/East, and this is given by:

$$P_{ws}^{1-\varepsilon} = N_s\left[\left(\frac{\tau\omega}{\alpha}\right)^{1-\varepsilon}V_s(a_{X,ws})\right]. \quad \text{(C.52)}$$

Bear in mind that $G_i(a) = \left(\frac{a}{\bar{a}_i}\right)^{\kappa}$ and $V_i(a) = \frac{\kappa}{\kappa-\varepsilon+1}\left(\frac{a^{\kappa-\varepsilon+1}}{\bar{a}_i^{\kappa}}\right)$, as these are from the Pareto distribution.

We have thus augmented the equilibrium system in the three-country model with two additional equations (C.48) and (C.52), corresponding to two additional endogenous variables, $a_{X,ws}$ and $P_{ws}^{1-\varepsilon}$. Note that relative to our baseline model, we have introduced only one new exogenous parameter, $f_{X,ws}$, in this extension. Intuitively, $f_{X,ws}$ governs the extent to which firms in West/East are shielded from the direct competition posed by Southern exporters. Also, note that as $f_{X,ws} \longrightarrow \infty$, we have $a_{X,ws} \longrightarrow 0$; this extension with Southern exporting thus nests the baseline model from the main paper. Intuitively, if $f_{X,ws}$ is high, exporting from South to West/East is difficult except for the very most productive Southern firms, and this limits the extent to which Southern varieties can compete in the markets in West/East. In such a situation, the effects of host-country financial development would clearly be similar to what we have derived for our baseline model.

Computational examples. The comparative statics of the above extension are cumbersome to study analytically, in large part because the equilibrium for the Southern FDI host country cannot be solved for in isolation from the feedback effect that arises from demand in West/East for South's exports. (Previously, the Southern equilibrium was pinned down by just two equations in Lemma 1.) We thus explore the behavior of the model with Southern exporting computationally; the Matlab code used for this exercise is available on request.

We focus first on the following parametrization which is based on the numerical example from the proof of Proposition 1: $R = 1.07$, $\varepsilon = 3.8$, $L_n = L_s = 1$, $f_D = 0.2$, $f_X = 0.15$, $f_I = 4$, $f_S = 0.1$, $f_{En} = f_{Es} = 1$, $\tau = 1.3$, $\omega = 0.7$, $\bar{a}_N = \bar{a}_S = 25$, $\kappa = 4$, $\delta = 0.1$, and $\mu = 0.5$. We work with a more moderate initial value of $\eta = 0.5$ and set $\phi = 0.75$, which respects the requirement that $R\frac{\phi}{1-\phi} > \frac{\eta}{1-\eta}$. We set $f_{X,ws} = 3$ to lie between the export fixed cost faced by firms headquartered in West/East, f_X, and the FDI fixed cost, f_I. With these parameter values, we obtain $a_D = 13.43$, $a_{XN} = 11.45$, $a_{XS} = 9.41$ and $a_I = 6.43$, as well as $a_S = 19.38$ and $a_{X,ws} = 5.38$.

This clearly satisfies the desired ordering of the productivity cut-offs in both West/East and South. Moreover, we obtain:

- $\frac{d}{d\eta}\ln HOR(a) = -0.144 < 0$, $\frac{d^2}{d\eta d\phi}\ln HOR(a) = 2.762 > 0$, $\frac{d}{d\eta}\ln PLA(a) = \frac{d}{d\eta}\ln RET(a) = 0.059 > 0$, and $\frac{d^2}{d\eta d\phi}\ln PLA(a) = \frac{d^2}{d\eta d\phi}\ln RET(a) = 20.159 > 0$;
- $\frac{d}{d\eta}\frac{HOR(a)}{TOT(a)} = \frac{d}{d\eta}\frac{HOR}{TOT} = -0.048 < 0$, $\frac{d^2}{d\eta d\phi}\frac{HOR(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} = -4.084 < 0$, $\frac{d}{d\eta}\frac{PLA(a)}{TOT(a)} = \frac{d}{d\eta}\frac{PLA}{TOT} = \frac{d}{d\eta}\frac{RET(a)}{TOT(a)} = \frac{d}{d\eta}\frac{RET}{TOT} = 0.024 > 0$, and $\frac{d^2}{d\eta d\phi}\frac{PLA(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} = \frac{d^2}{d\eta d\phi}\frac{RET(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{RET}{TOT} = 2.042 > 0$; and
- $\frac{d}{d\eta}\ln N_n G_n(a_I) = -0.062 < 0$, $\frac{d^2}{d\eta d\phi}\ln N_n G_n(a_I) = 21.270 > 0$, $\frac{d}{d\eta}\ln HOR = -0.157 < 0$, $\frac{d^2}{d\eta d\phi}\ln HOR = 13.092 > 0$, $\frac{d}{d\eta}\ln PLA = \frac{d}{d\eta}\ln RET = 0.046 > 0$, and $\frac{d^2}{d\eta d\phi}\ln PLA = \frac{d^2}{d\eta d\phi}\ln RET = 30.489 > 0$.

Even with the introduction of Southern exporting, the comparative statics with respect to financial development η remain broadly in line with the statement of Proposition 1 of our baseline model (in the absence of host-country financing): An improvement in host-country financial development leads to a decrease in affiliate sales to the local market, and an increase in return sales to West and platform sales to East, both in terms of the level of an individual affiliate's sales and in their shares out of total sales. In addition, the competition effect in the host-country market leads to a decrease in the number of affiliates on the extensive margin, as well as a reduction in the levels of aggregate horizontal sales. The cross-derivative effects with respect to external finance dependence ϕ moreover confirm that for the sales shares by market destination, these effects are stronger for industries that require a greater reliance on external capital. (The positive cross-derivative effect on affiliate horizontal sales is admittedly not in line with the predictions of the proposition and is harder to interpret. Likewise, the rise in aggregate platform and return sales in response to an improvement in financial development points to subtle general equilibrium effects, in which it is possible for the rise in aggregate demand levels per variety in West/East to dominate the negative effect from a decrease in the number of multinationals.)

We turn next to discuss the case where multinationals require host-country financing, as this serves to further illustrate the role of $f_{X,ws}$ in governing the strength of the feedback effect from Southern exporting. Recall that the expression for the FDI cut-off is now replaced by:

$$\tilde{a}_I^{1-\varepsilon} = \frac{1}{\eta}a_I^{1-\varepsilon}, \tag{C.53}$$

where a_I is given by (A.18). The extent of host-country financial development now has a direct effect on the FDI cut-off, while all other equations in the equilibrium system remain unchanged.

We first retain the above parameterization with $f_{X,ws} = 3$ and compute the equilibrium for the model with host-country financing *and* Southern exporting. For this, we obtain: $a_D = 13.53$, $a_{XN} = 11.53$, $a_{XS} = 9.44$ and $\tilde{a}_I = 5.46$, as well as $a_S = 19.44$ and $a_{X,ws} = 5.42$, so that the equilibrium ordering of the cut-offs is preserved. We also have:

- $\frac{d}{d\eta}\ln HOR(a) = -0.186 < 0$, $\frac{d^2}{d\eta d\phi}\ln HOR(a) = -0.394 < 0$, $\frac{d}{d\eta}\ln PLA(a) = \frac{d}{d\eta}\ln RET(a) = -0.062 < 0$, and $\frac{d^2}{d\eta d\phi}\ln PLA(a) = \frac{d^2}{d\eta d\phi}\ln RET(a) = -0.113 < 0$;
- $\frac{d}{d\eta}\frac{HOR(a)}{TOT(a)} = \frac{d}{d\eta}\frac{HOR}{TOT} = -0.029 < 0$, $\frac{d^2}{d\eta d\phi}\frac{HOR(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} = -0.066 < 0$, $\frac{d}{d\eta}\frac{PLA(a)}{TOT(a)} = \frac{d}{d\eta}\frac{PLA}{TOT} = \frac{d}{d\eta}\frac{RET(a)}{TOT(a)} = \frac{d}{d\eta}\frac{RET}{TOT} = 0.015 > 0$, and $\frac{d^2}{d\eta d\phi}\frac{PLA(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} = \frac{d^2}{d\eta d\phi}\frac{RET(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{RET}{TOT} = 0.033 > 0$; and

- $\frac{d}{d\eta}\ln N_n G_n(a_I) = 2.624 > 0$, $\frac{d^2}{d\eta d\phi}\ln N_n G_n(a_I) = -0.200 < 0$, $\frac{d}{d\eta}\ln HOR = 0.565 > 0$, $\frac{d^2}{d\eta d\phi}\ln HOR = -0.329 < 0$, $\frac{d}{d\eta}\ln PLA = \frac{d}{d\eta}\ln RET = 0.690 > 0$ and $\frac{d^2}{d\eta d\phi}\ln PLA = \frac{d^2}{d\eta d\phi}\ln RET = -0.048 < 0$.

Observe in particular that the competition effect is still evident from the response of the sales share to increases in host-country financial development η; this effect is moreover evident too in the differential response of the sales shares with respect to external finance dependence ϕ. In this version of the model where MNCs require host-country financing, we moreover find that an improvement in financial development induces an expansion on the extensive margin of affiliate presence and aggregate sales to each destination (summed across all affiliates), as was the case in the main paper. Interestingly though, we also see that individual affiliate sales to each destination market decrease. With Southern exporting, an increase in η raises the competition that Southern firms pose both in the local market, as well as in the Western/Eastern markets; as this numerical example illustrates, this can happen to such a degree that individual affiliate sales to all three destination markets fall in levels.

C.3 Model Extension: Endogenous host-country wages

The extended model with endogenous Southern wages is the case where $\mu = 1$ in our baseline model; this shuts down the homogenous-good sector. To keep the discussion of the mechanisms here as clean as possible, we focus on the case in which $K = 1$. We thus have exactly one differentiated-varieties sector that comprises the entire production side of the economy. The equilibrium is pinned down by the system of equations for the baseline model (setting $\mu = 1$), together with the additional labor market clearing condition (C.54) which we reproduce below:

$$\begin{aligned} L_s &= N_s A_{ss}\left(\frac{\omega}{\alpha}\right)^{-\varepsilon} V_s(a_S) + N_s R f_S G_s(a_S) + \delta N_s f_{Es} \\ &\quad + 2N_n\left[A_{ww}\tau\left(\frac{\tau\omega}{\alpha}\right)^{-\varepsilon} + A_{ew}\tau\left(\frac{\tau\omega}{\alpha}\right)^{-\varepsilon} + A_{sw}\left(\frac{\omega}{\alpha}\right)^{-\varepsilon}\right] V_n(a_I). \end{aligned} \tag{C.54}$$

In particular, this last equation serves to pin down the additional endogenous variable, i.e., the Southern wage ω, in the equilibrium system.

Computational examples. We base this discussion around the parameter values earlier seen in the proof of Proposition 1 and the prior exercise with Southern exporting: $R = 1.07$, $\varepsilon = 3.8$, $L_n = L_s = 1$, $f_D = 0.2$, $f_X = 0.15$, $f_I = 4$, $f_S = 0.1$, $f_{En} = f_{Es} = 1$, $\tau = 1.3$, $\bar{a}_N = \bar{a}_S = 25$, $\kappa = 4$, and $\delta = 0.1$. We work with a more moderate initial value of $\eta = 0.5$ and set $\phi = 0.75$, which respects the requirement that $R\frac{\phi}{1-\phi} > \frac{\eta}{1-\eta}$. We consider the baseline model without the financing effect, to focus on how endogenous host-country wages would affect the competition effect, although it should be clear that these implications would carry over even in the richer version of the model with the financing effect.

With the parameter values listed above, the equilibrium wage ω in South is pinned down endogenously and is equal to 0.87. (The industry cut-offs in West/East are respectively: $a_D = 14.61$, $a_{XN} = 12.45$, $a_{XS} = 11.33$ and $a_I = 4.50$.) Moreover, in response to a small change in η, we obtain:

- an increase in the Southern wage, $\frac{d}{d\eta}\ln\omega = 0.032$; and
- $\frac{d}{d\eta}\frac{HOR(a)}{TOT(a)} = \frac{d}{d\eta}\frac{HOR}{TOT} = -0.055 < 0$, and $\frac{d}{d\eta}\frac{PLA(a)}{TOT(a)} = \frac{d}{d\eta}\frac{PLA}{TOT} = \frac{d}{d\eta}\frac{RET(a)}{TOT(a)} = \frac{d}{d\eta}\frac{RET}{TOT} = 0.027 > 0$, as well as: $\frac{d^2}{d\eta d\phi}\frac{HOR(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} = -0.114 < 0$, and $\frac{d^2}{d\eta d\phi}\frac{PLA(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} = \frac{d^2}{d\eta d\phi}\frac{RET(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{RET}{TOT} = 0.057 > 0$.

This baseline set of parameters therefore yields implications for the sales shares that are in line with the competition effect.

We have moreover explored how this pattern of response of the sales shares is affected by shifts in the size of the labor force of West/East relative to that in South, this being a key underlying primitive that could affect the magnitude of the equilibrium wage response that one sees following an improvement in η. It turns out that the competition effect is little affected by shifts in L_n. For example, increasing L_n from its initial value of 1 to a higher value of 10 (holding all other parameters constant) yields:

- $\frac{d}{d\eta}\frac{HOR(a)}{TOT(a)} = \frac{d}{d\eta}\frac{HOR}{TOT} = -0.054 < 0$, and $\frac{d}{d\eta}\frac{PLA(a)}{TOT(a)} = \frac{d}{d\eta}\frac{PLA}{TOT} = \frac{d}{d\eta}\frac{RET(a)}{TOT(a)} = \frac{d}{d\eta}\frac{RET}{TOT} = 0.027 > 0$, as well as: $\frac{d^2}{d\eta d\phi}\frac{HOR(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{HOR}{TOT} = -21.390 < 0$, and $\frac{d^2}{d\eta d\phi}\frac{PLA(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{PLA}{TOT} = \frac{d^2}{d\eta d\phi}\frac{RET(a)}{TOT(a)} = \frac{d^2}{d\eta d\phi}\frac{RET}{TOT} = 10.695 > 0$.

C.4 Model Extension: Multiple host countries

We show how our model can be extended to speak to a comparison across multiple host countries with different levels of financial development. The effects that we have highlighted, particularly the competition effect, are thus relevant for understanding the cross-sectional variation in multinational activity as well.

It should be clear that the number of combinatorial possibilities for a given firm's export-versus-FDI decision increase considerably when there is more than one possible host country. The approach we take seeks to be as parsimonious as possible for the sake of tractability. For ease of exposition, we consider the case where $K = 1$, namely where there is only a single differentiated-varieties sector, but it should be clear that the argument we develop here holds for each sector if $K > 1$; we thus omit any sector superscripts in what follows. We consider a setup that is identical to our baseline model from Section 2.1, except that there are now two Southern countries that can host multinationals that emerge from West/East. We refer to the Southern countries as '$s1$' and '$s2$', these being the subscripts that we use to index the two countries. Both Southern countries are identical in all respects except their level of financial development. In particular, the nominal wage is $\omega < 1$ in both $s1$ and $s2$, and this is pinned down by the marginal product of labor in the homogeneous goods sector in each country. Each Southern country also has a differentiated-varieties industry, with firms that are heterogeneous in their productivity levels. The structure of this industry in both countries is identical to that for South in the baseline model, except that the level of financial development in $s1$ is higher than that in $s2$. In other words, we assume that $0 < \eta_2 < \eta_1 < 1$ without loss of generality.

On the demand side, we maintain the baseline assumption that consumers in West/East only desire differentiated-varieties from the Western and Eastern industries (as well as the homogeneous good). The utility function for consumers from $n = w, e$ is thus given once again by:

$$U_n = y_n^{1-\mu}\left(\sum_{j\in\{e,w\}}\int_{\Omega_{nj}} x_{nj}(a)^{\alpha}\ dG_j(a)\right)^{\frac{\mu}{\alpha}}, \tag{C.55}$$

where $0 < \alpha, \mu < 1$. On the other hand, consumers in each Southern country derive utility from Western/Eastern varieties, as well as from the varieties of their respective domestic industries; for simplicity, they do not consume the varieties made by the other Southern country. In other words, utility in each si, where $i = 1, 2$, is given by:

$$U_{si} = y_{si}^{1-\mu}\left(\sum_{j\in\{e,w,si\}}\int_{\Omega_{si,j}} x_{si,j}(a)^{\alpha}\ dG_j(a)\right)^{\frac{\mu}{\alpha}}. \tag{C.56}$$

Solving the standard utility maximization problem then implies the following expressions for the aggregate market demand levels:

$$A_{ww} = A_{ee} = A_{ew} = A_{we} = \frac{\mu L_n}{P_{ww}^{1-\varepsilon} + P_{we}^{1-\varepsilon}}, \quad \text{and} \tag{C.57}$$

$$A_{si,w} = A_{si,e} = A_{si,si} = \frac{\mu \omega L_s}{P_{si,si}^{1-\varepsilon} + 2P_{si,w}^{1-\varepsilon}}. \tag{C.58}$$

Note that $A_{si,si}$ is now the demand level in country si for its domestic differentiated-varieties, while $A_{si,w}$ and $A_{si,e}$ are the corresponding demand levels for the varieties from West and East respectively. From (C.58), these are functions in particular of the ideal price indices for country-si varieties consumed domestically, $P_{si,si}$, and for Western/Eastern varieties consumed in si, $P_{si,w}$.

We examine first the equilibria in the two Southern differentiated-varieties industries. Following the industry structure from our baseline model, the productivity cut-off for domestic entry in each Southern country, which we denote by $a_{Si}^{1-\varepsilon}$ for $i = 1, 2$, is given by:

$$a_{Si}^{1-\varepsilon} = \frac{1}{\eta_i} \frac{R\phi f_S \omega}{(1-\alpha) A_{si,si} (\omega/\alpha)^{1-\varepsilon}}. \tag{C.59}$$

Analogously, we can write down the free entry condition in each country si ($i = 1, 2$) as:

$$f_{Es}\omega = \frac{1}{\delta}\left[(1-\alpha)A_{si,si}\left(\frac{\omega}{\alpha}\right)^{1-\varepsilon} V_s(a_{S,i}) - (R\phi + (1-\phi)) f_S \omega G_s(a_{S,i})\right]. \tag{C.60}$$

Inspecting (C.59) and (C.60), it follows as a quick corollary of Lemma 1 that when $\eta_1 > \eta_2$, we must have $A_{s1,s1} < A_{s2,s2}$. This is intuitive since *ceteris paribus*, the country with the higher level of financial development would facilitate more entry by local firms, so that the aggregate demand level faced by each firm would be lower. From (C.58), we thus have: $A_{s1,w} < A_{s2,w}$.

We turn next to the differentiated-varieties sector in West/East. In keeping with the spirit of our baseline model, we focus on situations in which if a firm from West (likewise East) decides to undertake FDI in either one of the Southern countries, then that Southern facility will be used as the global production center for that firm from which all four markets will be serviced, including the other Southern country. (In particular, we assume that the fixed cost of FDI, f_I, is sufficiently large so that the multinational will never seek to establish more than one foreign affiliate.)

For ease of exposition, we adopt the perspective of a firm headquartered in West; the situation for a firm from East is entirely symmetric. Suppose that this firm has productivity $1/a$. If this Western firm undertakes FDI in host country si, where $i \in \{1, 2\}$, then the horizontal, platform and return sales of its affiliate in si are given explicitly by:

$$HOR_{si}(a) = A_{si,w} (a\omega/\alpha)^{1-\varepsilon}, \tag{C.61}$$

$$PLA_{si}(a) = (A_{e,w} + A_{sj,w}) (\tau a\omega/\alpha)^{1-\varepsilon}, \quad \text{and} \tag{C.62}$$

$$RET_{si}(a) = A_{w,w} (\tau a\omega/\alpha)^{1-\varepsilon}, \tag{C.63}$$

where $j \in \{1, 2\}$ and $j \neq i$. (In other words, we use the subscript 'sj' to refer to variables relevant to the Southern country where the firm does not undertake FDI.) From (C.68)-(C.70), the corresponding destinationspecific shares out of total sales are therefore:

$$\frac{HOR_{si}(a)}{TOT_{si}(a)} = \left(1 + \frac{\tau^{1-\varepsilon}(A_{ww} + A_{e,w} + A_{sj,w})}{A_{si,w}}\right)^{-1}, \tag{C.64}$$

$$\frac{PLA_{si}(a)}{TOT_{si}(a)} = \left(1 + \frac{A_{si,w} + \tau^{1-\varepsilon}A_{w,w}}{\tau^{1-\varepsilon}(A_{e,w} + A_{sj,w})}\right)^{-1}, \text{ and} \tag{C.65}$$

$$\frac{RET_{si}(a)}{TOT_{si}(a)} = \left(1 + \frac{A_{si,w} + \tau^{1-\varepsilon}(A_{e,w} + A_{sj,w})}{\tau^{1-\varepsilon}A_{w,w}}\right)^{-1}. \tag{C.66}$$

(Note that: $TOT_{si}(a) = HOR_{si}(a) + PLA_{si}(a) + RET_{si}(a)$.) Observe that these sales shares are identical across all firms, as they do not depend on a. Hence, the expressions in (C.64)-(C.66) are also equal to the horizontal, platform and return sales shares aggregating across all multinational affiliates from West that are present in country si.

We now make use of the fact that $A_{s1,w} < A_{s2,w}$ when $\eta_1 > \eta_2$. Also, bear in mind that each firm from West takes the aggregate demand levels, $A_{w,w}$, $A_{e,w}$, $A_{s1,w}$ and $A_{s2,w}$, as given. In particular, these demand levels are unaffected by the decision of the firm to undertake FDI in either $s1$ or $s2$. Applying some straightforward algebra on (C.64)-(C.66), it immediately follows that: $\frac{HOR_{s1}(a)}{TOT_{s1}(a)} < \frac{HOR_{s2}(a)}{TOT_{s2}(a)}$, $\frac{RET_{s1}(a)}{TOT_{s1}(a)} > \frac{RET_{s2}(a)}{TOT_{s2}(a)}$ and $\frac{PLA_{s1}(a)}{TOT_{s1}(a)} > \frac{PLA_{s2}(a)}{TOT_{s2}(a)}$. In words, we recover the essence of the competition effect in a cross-country comparison across host countries. Where financial development is higher, the local market is a more competitive environment, so that the share of horizontal sales is lower, while the return and platform sales shares are higher. With this multiple host country setup, the implications of host-country financial development for the sales shares of MNC affiliates thus continue to hold in the cross-section. (Incidentally, in this extension, we also break the symmetry in the magnitudes of the return and platform sales shares, since platform sales would also include sales to the other Southern country.)

We now turn to the task of comparing affiliate and aggregate sales levels across the different host countries. As mentioned in Section B.4, this requires that we introduce more structure to the model: For the affiliate-level comparison to be one that "holds all else constant", the model should allow for different multinationals with the same productivity level $1/a$ to potentially choose to locate in either $s1$ or $s2$. There are various modeling strategies for achieving this, and we present one such possibility here based on allowing for idiosyncratic realizations of profit shocks from locating in each respective host country.

Consider first an initial setting in which MNCs do not require host-country financing. Western firms that are contemplating FDI now face both a systematic and a stochastic component to the profits they will earn from locating in either host country. The systematic component is known in advance, and is equal to their sales less variable and fixed costs from basing production in the host country in question. However, there is now an additive stochastic component to these profits, denoted by ν_{s1} and ν_{s2} in the respective host countries; one can view these as firm-specific idiosyncratic costs that are ex-ante uncertain, the precise values of which are only revealed after the firm has made a decision to pursue FDI. To be clear on the timing of events, a Western firm first obtains its productivity draw $1/a$, on the basis of which it makes an irreversible decision whether or not to become a multinational. If it should choose to pursue FDI, it then observes the stochastic draws of ν_{s1} and ν_{s2}, from which it decides which of $s1$ or $s2$ to locate its affiliate in. Firms that choose not to engage in FDI can either exit, remain purely domestic, or service the foreign markets through exporting, although for the purposes of this extension, the details of these options are less important.[6]

[6]The irreversibility of the FDI decision could be justified if there were a component of ν_{s1} and ν_{s2} that needs to be incurred as

For a firm that chooses FDI, the realized profits from locating its production affiliate in $s1$ and $s2$ are given respectively by:

$$\pi_{I,s1}(a) = (1-\alpha)\left(A_{s1,w} + \tau^{1-\varepsilon}(A_{w,w} + A_{e,w} + A_{s2,w})\right)\left(\frac{a\omega}{\alpha}\right)^{1-\varepsilon} - f_I + \nu_{s1}$$
$$\pi_{I,s2}(a) = (1-\alpha)\left(A_{s2,w} + \tau^{1-\varepsilon}(A_{w,w} + A_{e,w} + A_{s1,w})\right)\left(\frac{a\omega}{\alpha}\right)^{1-\varepsilon} - f_I + \nu_{s2}.$$

In the above, we specify ν_{s1} and ν_{s2} to be iid shocks drawn from a standard Gumbel distribution. Using the well known properties of the extreme-value distribution, some simple algebra leads to the following expression for the probability, p_{s1}, that $\pi_{I,s1}(a) > \pi_{I,s2}(a)$ and hence that $s1$ will be chosen as the host country:

$$p_{s1} = \frac{\exp\{(1-\alpha)\left(A_{s1,w} + \tau^{1-\varepsilon}A_{s2,w}\right)(a\omega/\alpha)^{1-\varepsilon}\}}{\exp\{(1-\alpha)\left(A_{s1,w} + \tau^{1-\varepsilon}A_{s2,w}\right)(a\omega/\alpha)^{1-\varepsilon}\} + \exp\{(1-\alpha)\left(A_{s2,w} + \tau^{1-\varepsilon}A_{s1,w}\right)(a\omega/\alpha)^{1-\varepsilon}\}}. \quad \text{(C.67)}$$

Since $A_{s1,w} < A_{s2,w}$, we can deduce that $A_{s1,w} + \tau^{1-\varepsilon}A_{s2,w} < A_{s2,w} + \tau^{1-\varepsilon}A_{s1,w}$, and hence that $p_{s1} < 1 - p_{s1}$. There is thus a larger probability that the profits from locating in $s2$ will exceed the profits from locating in $s1$, since $s1$ features a more competitive goods market by virtue of its higher level of financial development. By the law of large numbers, a fraction p_{s1} (respectively, $p_{s2} \equiv 1 - p_{s1}$) of Western firms with productivity $1/a$ will choose $s1$ (respectively, $s2$) as their host country. In turn, a Western firm with productivity $1/a$ will choose to become a multinational if it finds that its expected profits from undertaking FDI, given by $E[\max\{\pi_{I,s1}(a), \pi_{I,s2}(a)\}]$, exceed that from instead retaining production at home and exporting from there to all the other foreign markets. Note that the preceding expectation will have to be evaluated over the distribution of the iid profit shocks, ν_{s1} and ν_{s2}. We do not work out this expectation explicitly, as it suffices for our purposes that this will yield a unique cut-off value which we call $a_{I,twoS}$.[7] In other words, the most productive Western firms, with a productivity draw $1/a > 1/a_{I,twoS}$, will then venture into FDI, and will decide on either $s1$ or $s2$ for their host country after observing their realizations of ν_{s1} and ν_{s2}.

We now compare the sales levels of two distinct affiliates with the same productivity $1/a$ that are nevertheless located in different host countries. From equations (C.61)-(C.63), and the fact that $A_{s1,w} < A_{s2,w}$, it follows immediately that: $HOR_{s1}(a) < HOR_{s2}(a)$, $PLA_{s1}(a) > PLA_{s2}(a)$ and $RET_{s1}(a) = RET_{s2}(a)$. At the affiliate level, we therefore recover the implication that the horizontal sales level will be lower and the platform sales level higher in the host country where financial conditions are better. (Admittedly, the mapping of predictions into the cross-section is not perfect, as we now have that the level of return sales to West would be identical for the affiliates in the two host countries.)

The extra structure of the distributional assumption on the ν_{si}'s further allows us to compare the sales levels aggregated across multinational affiliates in the two host countries. Based on our discussion above, the measure of affiliates in country si ($i = 1, 2$) is given precisely by: $p_{si}N_nG_n(a_{I,twoS})$. We can also write down the aggregate levels of horizontal, platform and return sales in each si as:

$$HOR_{si} = p_{si}A_{si,w}\left(\omega/\alpha\right)^{1-\varepsilon}V_n(a_{I,twoS}), \quad \text{(C.68)}$$
$$PLA_{si} = p_{si}(A_{e,w} + A_{sj,w})\left(\tau\omega/\alpha\right)^{1-\varepsilon}V_n(a_{I,twoS}), \quad \text{and} \quad \text{(C.69)}$$
$$RET_{si} = p_{si}A_{w,w}\left(\tau\omega/\alpha\right)^{1-\varepsilon}V_n(a_{I,twoS}), \quad \text{(C.70)}$$

a cost when these stochastic shocks are first observed. For example, one could think of the ν_{si}'s as a learning cost to discover one's true profitability in each host country, and that a part of these costs becomes sunk once the realizations of ν_{s1} and ν_{s2} are learnt.

[7] To be fully precise, $a_{I,twoS}$ will be pinned down in conjunction with a free-entry condition for West.

Since $p_{s1} < p_{s2}$, we immediately have that the measure of affiliates is lower in the more financially-developed host, $s1$. Moreover, from (C.63), it is clear that $RET_{s1} < RET_{s2}$. Next, using the additional fact that $A_{s1,w} < A_{s2,w}$, we have from (C.61) that $HOR_{s1} < HOR_{s2}$. Finally, to compare PLA_{s1} and PLA_{s2}, observe from (C.67) that:

$$\frac{p_{s1}}{p_{s2}} = \frac{\exp\{(1-\alpha)\left(A_{s1,w}+\tau^{1-\varepsilon}A_{s2,w}\right)(a\omega/\alpha)^{1-\varepsilon}\}}{\exp\{(1-\alpha)\left(A_{s2,w}+\tau^{1-\varepsilon}A_{s1,w}\right)(a\omega/\alpha)^{1-\varepsilon}\}} = \frac{\exp\{(1-\alpha)A_{s1,w}(1-\tau^{1-\varepsilon})(a\omega/\alpha)^{1-\varepsilon}\}}{\exp\{(1-\alpha)A_{s2,w}(1-\tau^{1-\varepsilon})(a\omega/\alpha)^{1-\varepsilon}\}} < \frac{A_{s1,w}}{A_{s2,w}},$$

where the last inequality comes from applying the fact that: (i) $\exp\{x\}/x$ is an increasing function in x for all $x > 1$; and (ii) $A_{s1,w} < A_{s2,w}$. (Note that we need to ensure through a suitable normalization that $(1-\alpha)\left(A_{s1,w}+\tau^{1-\varepsilon}A_{s2,w}\right)(a\omega/\alpha)^{1-\varepsilon}$ and $(1-\alpha)\left(A_{s2,w}+\tau^{1-\varepsilon}A_{s1,w}\right)(a\omega/\alpha)^{1-\varepsilon}$ both exceed 1, so that property (i) can be applied. This can be achieved by assuming that the labor endowment in each host country is sufficiently big.) From the above, we have that: $p_{s1}A_{s2,w} < p_{s2}A_{s1,w}$, which together with (C.62) implies that $PLA_{s1} < PLA_{s2}$. In sum, we find that comparing the two FDI hosts, the country with the higher level of financial development features fewer affiliates and lower aggregate sales levels; this provides the analogue to part (iii) of Proposition 1.

Last but not least, we briefly discuss the case where multinationals require host country financing. Observe that the expressions for the sales shares in (C.64)-(C.66) and for the sales levels of individual affiliates in (C.61)-(C.63) remain valid even when MNCs seek local financing, as long as the affiliates being compared are both able to secure this financing from the respective host country institutions. The same arguments as above can then be applied to show that $s1$ will still be a more competitive market environment than $s2$, so that $A_{s1,w} < A_{s2,w}$. One can then quickly see that the following comparisons still hold: $\frac{HOR_{s1}(a)}{TOT_{s1}(a)} < \frac{HOR_{s2}(a)}{TOT_{s2}(a)}$, $\frac{RET_{s1}(a)}{TOT_{s1}(a)} > \frac{RET_{s2}(a)}{TOT_{s2}(a)}$, $\frac{PLA_{s1}(a)}{TOT_{s1}(a)} > \frac{PLA_{s2}(a)}{TOT_{s2}(a)}$ and $HOR_{s1}(a) < HOR_{s2}(a)$. These cross-sectional implications are consistent with parts (i) and (ii) of Proposition 2.

With host-country financing, the analysis for aggregate measures of multinational activity is in general more complicated in terms of the cases that would need to be enumerated. For example, it is possible that some prospective multinationals would be productive enough to receive funding in country $s1$, but not in $s2$. However, a clear comparison can nevertheless be made in the limiting case where $\eta_2 \longrightarrow 0$. In this situation, the cost of default would approach zero in $s2$. In the limit, there would therefore be no affiliates in $s2$, although there would be a positive measure in $s1$. The number of multinational affiliates, as well as the aggregate levels of horizontal, platform and return sales, would clearly be higher in the more financially-developed host country $s1$ than in $s2$. In sum, when host-country financing is required, the qualitative prediction that the aggregate level of multinational activity would be higher in $s1$ is preserved when financial development in $s2$ is sufficiently low.